THE STUDY OF MAGNETIZATION PROCESSES

USING MONTE CARLO METHODS

KOVÁCS KATALIN

PHD THESIS

PRESENTED AT FACULTY OF PHYSICS,

BABEŞ-BOLYAI UNIVERSITY

SCIENTIFIC ADVISOR: PROF. DR. NÉDA ZOLTÁN

2007
Contents

Abstract ......................................................... 1
Motivation ....................................................... 2
Acknowledgments ............................................... 3

1 Basic Concepts ............................................... 4

2 Review of Recent Experimental Measurements Regarding Magnetization Phenomena in Disordered Ferromagnets ................................................. 10
2.1 Some Relevant Experiments .................................. 11

3 Review of Theoretical Models Aimed to Explain Magnetization Phenomena in Ferromagnetic Materials .................................................. 20
3.1 The ABBM Model .............................................. 23
3.2 Spin Models ................................................... 25
3.2.1 Random Field Ising Model ............................... 25
3.2.2 Random Bond Ising Model ............................... 31
3.2.3 Random Anisotropy Ising Model ......................... 36
3.3 Domain Wall Models ........................................... 40
3.3.1 The UMM Model .......................................... 40
3.3.2 Narayan’s Model .......................................... 43
3.3.3 Infinite- and Long Range Domain Wall Model ............ 43
3.4 Stochastic Resonance in Magnetization Models ................. 48

4 Spring-block Model ............................................ 51
4.1 Presentation of the Spring-block Model ....................... 51
4.1.1 Dynamics ................................................ 55
4.1.2 Influence of the free parameters ........................ 57
4.2 Simulation Results ............................................ 59
4.3 Disorder-driven Phase Transition in the Spring-block Model .............. 62
4.4 Overview of the Spring-block Model ........................ 68
## 5 Spin Hamiltonian Model with Random Pinning Centers

5.1 Presentation of the Spin Hamiltonian Model

5.1.1 Hamiltonian

5.1.2 Dynamics

5.1.3 Influence of the free parameters

5.2 Simulation results

5.3 Stochastic Resonance in the Spin Hamiltonian Model

5.3.1 Results for Stochastic Resonance

5.4 Overview of the Spin Hamiltonian Model with Random Pinning Centers

## 6 Final Conclusions

References
Abstract

In this thesis magnetization phenomena and in special the magnetic Barkhausen noise is studied.

A simple one-dimensional spring-block type model is introduced and presented. The model is based on a mechanical analogy and its aim is to explain the basic features of domain-wall dynamics and magnetization processes in ferromagnetic materials. Despite its simplicity the spring-block model yields the realistic shape of hysteresis loops with Barkhausen jumps, it is able to reproduce the interesting phenomenon of disorder-induced phase transition, as well as the good statistics of Barkhausen noise. The results obtained by the model are in qualitative agreement with experiments.

Although, the spring-block model has many qualities, it is far from being perfect. Its main deficiencies are: the predefined number of magnetic domains, the lack of explicit mathematical description. In order to overcome these deficiencies, a spin Hamiltonian model was introduced. This is also one-dimensional model described by a Hamiltonian which contains all the interactions relevant in real ferromagnets: Ising-type exchange, pinning, interaction with the external field, demagnetization. The model is able to reproduce all the expected statistical properties of Barkhausen noise. In addition, there is a possibility to study the stochastic resonance in this model because it is suitable for Monte Carlo simulations.

The present thesis is structured as follows. In an introductory chapter the topic is presented in a general context, the phenomena under study are described and a number of concepts that will be used further in the thesis are clarified. The second and third chapters are dedicated to the review of the recent experimental and theoretical studies regarding Barkhausen noise and magnetization phenomena. The fourth and fifth chapters constitute the main original part of the thesis. In these chapters the original models are introduced and studied from several viewpoints. A significant part of these two chapters was already published in four peer-reviewed papers. Finally, in the last chapter a summary of the thesis and final conclusions are given.
Motivation

The aim of this work is to show the importance and value of very simple models in describing rather complex physical phenomena.

In this work it will be proven that two oversimplified one-dimensional toy-models are able to reproduce at least qualitatively the main features of hysteresis, domain-wall dynamics, avalanche-like behavior and all the related scaling properties.

The actual problem studied in this thesis is the magnetic Barkhausen noise and related magnetization phenomena. Two original models are introduced for exploring these phenomena and for reproducing their measured statistical properties.

Magnetization phenomena are interesting and worth to study both for their many practical applications and from the viewpoint of statistical physics. A practical reason would be that the measuring and analysis of the Barkhausen noise is the basis of a non-destructive and non-invasive material testing and control. Conceptually, understanding the statistical properties of such phenomena can lead to a deeper knowledge regarding the microscopic (or mesoscopic) dynamics in ferromagnetic materials.

The success of such simple models like those presented in this thesis indicates that the statistical properties of magnetization phenomena depend usually only on a small number of factors, like the ferromagnetic coupling, the amount of disorder in the system, demagnetization effects, temperature and external driving rate.
Acknowledgments

First of all I am very thankful to my scientific supervisor Dr. Néda Zoltán who helped me in every step in this work. Without his guidance, help, patience and brilliant ideas this thesis could not even come into existence.

Professor Yves Bréchet from Grenoble was the first who realized that the spring-block type model can be useful in modeling magnetization phenomena and Barkhausen noise. I thank Professor Bréchet for sharing his brilliant idea and for accepting our invitation to be a member of the committee at the public defense of this thesis.

I acknowledge the working community at the Babeș-Bolyai University for providing a pleasant working atmosphere and for their encouragement.

I acknowledge Nicolas Meyer from Institute Néel Grenoble France because he shared with me his experimental results obtained from numerous measurements on ferromagnetic materials. His results gave me the unique possibility to directly compare simulation results with the experimental findings. Beside this Nicolas was very helpful and answered me many questions.

I thank professor Vicsek Tamás and his research group from Eötvös Loránd University Budapest for their hospitality. They provided me with professional help and appropriate working conditions for three months from February to May 2007. This period was essential for me, a perfect opportunity to finalize the present thesis.

Finally I would like to thank my family and my friends for their care and encouragement. They were always beside me in the critical moments.

Due to funds from research grants CNCSIS 4/183 2005-2007 and CNCSIS 3/31 2005-2006 I received free access to remarkable computational power. This made simulations run faster and thus I could obtain results in due time. In addition due to these funds it was possible for me to participate on several conferences where a considerable part of this work was presented and discussed.
Chapter 1

Basic Concepts

In this very first chapter we discuss and define the basic concepts and terminology often used within the work.

**Hysteresis** is characteristic for many different physical systems. It is characterized by history dependent evolution when the system is driven out of equilibrium by an external forcing. The system usually does not follow instantaneously the external force (or field) applied on it, and the response involves the collective (avalanche-like) motion of many units (building blocks) which the system in case consists of. The evolution of such systems is also irreversible, thus the shape of the response function versus the forcing forms a loop. The measure of the irreversibility for such a process can be considered to be proportional with the area enclosed by the hysteresis loop.

Some examples of different physical systems showing hysteretic behavior are: (i) magnetic materials – especially ferromagnets; (ii) elastic materials; (iii) electrical hysteresis in ferroelectric materials; (iv) liquid-solid transition in the case of materials whose melting and freezing temperature are not equal\(^1\) or (v) the supercooling of fluids (see example references for the different occurrences of hysteresis in [1]).

**Barkhausen noise** (BN) belongs to the family of the so-called crackling noises [2]. It appears as a series of discrete and abrupt jumps in the magnetization whenever a ferromagnetic sample is placed under varying external magnetic field. Many systems exhibit crackling noise, for example: fluids invading porous materials, the dynamics of superconductors and superfluids can be considered as crackling noise, sound emitted during martensitic phase transitions, fluctuations on stock market, group decision-making,

---

\(^1\)An example is the agar which is an unbranched polysaccharide obtained from the cell walls of some species of red algae. It melts at 85° C and solidifies around 40° C.
fracture in disordered materials. In standard ferromagnets where magnetization is driven by the motion of domain walls, it is believed that the BN is a consequence of the fast movement of domain walls between pinning centers, which are either defects or impurities (see Fig. 1.1.).

The Barkhausen phenomenon is interesting from several points of view. From a practical side, by measuring the BN there is a possibility for non-destructive and non-invasive material testing and control. On the other hand, from a pure conceptual viewpoint, by studying the BN one might reach a better understanding of the complex dynamics of domain walls during magnetization processes in the presence of pinning centers opposing to the motion of magnetic walls. It is also a clear example for the dynamics of a system presenting collective pinning when a quenched disorder is present.

Since its discovery (1917) BN has been intensively studied. Numerous measurements were done to clarify the statistical properties of the BN [3, 4, 5, 6]. Regarding the nature of the Barkhausen noise (white noise, 1/f noise, or 1/f² noise) even the experimental results are sometimes incoherent with each other.

BN received a special attention in the context of self-organized criticality.

Bak, Tang and Wiesenfeld [7] introduced the concept of self-organized criticality (SOC) as a general framework to understand the wide occurrence of power-law distributions in nature.

Criticality is the state of systems near phase transitions. The critical region in the vicinity of a phase transition is interesting from the viewpoint of statistical physics because the derivatives of the free energy have singularities. This divergence causes that the system becomes self-similar and relevant quantities show power-law distribution. Usually, in equilibrium thermodynamics an external parameter needs to be carefully fine-tuned in...
order to drive the system near the critical point. Well known examples are the tuning of temperature in the case of melting/freezing transition or the tuning of the external magnetic field in the case of ferromagnetic/paramagnetic phase transition.

However, there are non-equilibrium (usually open) dissipative systems which self-organize unto a critical state without the fine-tuning of any external parameter. A definition SOC would be [8]: "SOC refers to a statistical steady state that is produced by processes with an infinite separation of time scales, and that exhibits scale invariance without further fine tuning." (G. Grinstein)

The archetype of self-organized criticality is the "sandpile". In the sandpile model grains are dropped on a lattice, they pile up until a specified height is reached, after which they fall on the neighboring sites, forming avalanches that carry sand from the top of the pile to the bottom. In this way avalanches propagate through the system until they fall out of the lattice boundaries. The slope of the pile remains around the (self-organized) critical slope.

BN studies raised the question whether this phenomenon is an experimental evidence of self-organized criticality since some of the ingredients of SOC were known to be potentially relevant to BN. In some cases, magnetization changes have been directly observed to occur via avalanche process in the domain topology [9]. These avalanches exhibit some scaling effects, at least over a narrow range of parameters, and their behavior has been described by a SOC model [10, 11]. There are, however, other approaches that put under doubt the relevance of the SOC concept to BN. O’Brien and Weissman [12] for example argued that the presumed $1/f$ nature of BN and the observed power-law distributions are not necessarily evidences of SOC, but rather the consequences of the scaling properties of quenched disorder in the material.

Vives and Planes [13] give a very accurate definition of disordered system and its dynamics: "Disordered systems can be viewed as described by a multidimensional energy landscape containing many local metastable states separated by large energy barriers. If thermal fluctuations are not operative, these energy barriers can only be overcome by modifying the external field which tilts the energy landscape. The system evolution thus proceeds through jumps from one metastable state to another metastable state and therefore, the field-response shows a discontinuous and apparent stochastic character. In magnetic systems, this jerky magnetization response corresponds to the so-called Barkhausen noise." Because such systems show practically the same behavior on large
lengthscales, their behavior should be independent of microscopic and macroscopic details of the system suggesting **universality**. Universality makes it possible that simple models can provide useful tools to explain crackling phenomena, and to give predictions on the studied system’s behavior. On the other hand simple models have a price: they won’t be able to make clear predictions about the way how the real microscopic parameters affect the behavior of the system under study on long length scales.

Many models [13, 14, 15] aimed to describe magnetization phenomena predicted the existence of **disorder-driven phase transition** in disordered magnetic systems.

In the vicinity of a first-order phase transition there are usually three characteristic time-scales: $\tau_a$ is the characteristic time-scale of the microscopic (atomic) response of the system; $\tau_{th}$ is the characteristic time-scale of the thermal fluctuations and $\tau_{dr}$ the characteristic time-scale of the external driving. Whenever the $\tau_a << \tau_{dr} << \tau_{th}$ relation holds between the three characteristic time-scales, we can encounter a fluctuationless or athermal first order phase transition. In contrast, for the conventional equilibrium phase transitions the $\tau_a << \tau_{th} << \tau_{dr}$ relation holds between the characteristic time-scales.

Well-known examples for fluctuationless phase-transitions are the athermal solid-solid diffusionless martensitic transitions [16] or the field-induced first-order phase transition in ferromagnetic systems. For many fluctuationless phase transitions hysteresis occurs even if the system is driven extremely slowly. This suggests that hysteresis is not of kinetic nature, but it is due to the quenched disorder present in these systems. A special type of fluctuationless phase transition occurs when the amount of quenched disorder is varied in some systems. Changing the amount of disorder will qualitatively change their behavior, so this type of fluctuationless phase transition is named disorder-driven phase transition. Whenever disorder-driven phase transition occurs, the system has history-dependent metastable evolution, hysteresis is present. There exists a critical amount of disorder at which the qualitative shape of the hysteresis loop changes from sharp to smooth and the phase transition becomes continuous. Studying an exchange-coupled $Co/CoO$-bilayer magnetic structure, recently Berger et al. [17] gave also experimental evidence for the theoretically predicted disorder-driven phase transition.

**Scaling hypothesis** is a very important concept in studying critical phenomena. The existence of a correlation length $\xi$, which diverges for an infinite system at the critical
point $X_c$ is presumed:

$$\xi \sim \left( \frac{X - X_c}{X_c} \right)^{-\nu},$$

(1.1)

where $X$ is the parameter which induces the critical behavior. This can be for example disorder in the case of random-field, random-anisotropy and random-bond models or temperature for the paramagnetic-ferromagnetic phase transition. The scaling hypothesis assumes that any other quantity $A$ exhibiting non-analytical behavior at the critical point will scale as:

$$A \sim \xi^\alpha \sim \left( \frac{X - X_c}{X_c} \right)^{-\nu \alpha}.$$  

(1.2)

For a finite system of size $L$ the scaling is changed, finite-size scaling corrections have to be applied. The critical point $X_c$ changes to:

$$\tilde{X}_L = \frac{X - X_c(L)}{X_c(L)},$$

(1.3)

and the general behavior of a quantity $A$ near the critical point:

$$A \sim L^\alpha F_A(L^{1/\nu} \tilde{X}_L),$$

(1.4)

where $F_A$ is a scaling function with $L^{1/\nu} \tilde{X}_L$ scaling variable.

Some models [2, 18] use the renormalization group method introduced in the '70s by Kadanoff, Wilson and Fisher (for detailed study see for example [19]). The critical properties of a system do not depend on short-distance details, but only on the long-wavelength fluctuations. Renormalization group is a coarse-graining procedure when the irrelevant degrees of freedom are removed. When the laws governing the evolution of the system look the same after coarse-graining, it is called self-similarity, and it leads to universal scaling functions, very often power-laws.

**Stochastic resonance** (SR) is another phenomenon which is observed and well studied in many types of dynamical systems. Stochastic resonance occurs in a dynamical system when "the ordered response of a system with respect to weak input signals can be significantly increased by appropriately tuning the noise intensity to an optimal but nonvanishing value." (from Dynamics of Chaotic and Stochastic Systems by V. S. Anishchenko, A. B. Neiman, T. E. Vadivasova, V. V. Astakhov and L. Schimansky-Geier, Springer 2006) [20]. The effect requires three main ingredients: (i) energetic activation barrier, (ii) a weak
periodic signal, (iii) a source of noise [21]. Quantitatively: a sharp peak appears in the output power spectrum exactly at the input frequency, the signal-to-noise ratio (SNR) or the input/output cross-correlation measures have well-marked maxima at a finite noise level [20]. SR has been observed and studied in many different physical systems ranging from magnetic systems, ferromagnets, passive optical bistable systems, magneto-elastic ribbons, superconducting quantum interference devices (SQUID) to Brownian particles, chemical systems and biological systems like sensory neurons and ion channels or even in human psychophysics [22].
Chapter 2

Review of Recent Experimental Measurements Regarding Magnetization Phenomena in Disordered Ferromagnets

A wide variety of experimental data is available regarding the statistical properties of BN. All these agree that BN is characterized by several power-laws fitting the results for jump-size distribution, power spectrum, signal duration distribution, signal energy distribution. The numerical results for scaling exponents are, however widely different, sometimes contradictory. For this reason it is important to discuss the different experimental conditions and material properties before making direct comparison with any theory aimed to explain BN and account for its behavior or statistical properties quantitatively.

Most experiments were performed using an inductive method. Fig. 2.1. shows the sketch of a typical experimental setup. A signal generator provides current (with sinusoidal or triangular shape) for the driving solenoid, which drives magnetically the specimen through the hysteresis loop. The specimen is placed inside a second coil, called pickup coil. The Barkhausen signals (which correspond to jumps in the magnetization of the sample) were collected as induced voltage pulses via the pickup coil. The electric signal received from the pickup coil is amplified, digitalized, and analyzed.

The geometry of the sample is important [23], it influences the experimental results because the scaling is controlled by the rate of the external driving field and by the intensity of the demagnetizing field. This latter depends strongly on the shape of the
sample. Driving rate determines the exponents of jump-size distribution, and the cutoff is controlled by the demagnetizing field. Under well-defined experimental conditions [24, 25] the scaling exponents do not depend any more on the particular properties of the sample, which is the consequence of the universality. Measurements have to be taken only in the central part of the hysteresis loop around the coercive field because here domain wall motion is dominant in competition with domain nucleation, spin flip, or other types of magnetization processes. In this region magnetization process can be considered as stationary. Those experiments which want to provide linear magnetization work with triangular shape driving field instead of sinusoidal. Other experiments which investigate the variation of the response of the system under varying driving frequency use instead mostly sinusoidal signal.

2.1 Some Relevant Experiments

1. The work performed by Alessandro et al. [25] is considered to be the first accurate and reliable experiment and analysis of the obtained results. From the experiment the amplitude probability distribution of the induced magnetic flux rate $P_0(\dot{\Phi})$ ($\dot{\Phi} \equiv \frac{d\Phi}{dt}$) and the power spectrum $F(\omega)$ of the Barkhausen noise were extracted.

The material under study was a strip of nonoriented polycrystalline 3% SiFe of grain size $\sim 100\mu m$ and with dimensions $0.2 \times 0.01 \times (3.5 \times 10^{-4})m$. 
Data collecting

Measurements were done with the standard inductive method, and special care was taken to exclude spurious external magnetic fields, such as the Earth’s magnetization. The strip was subjected to a tensile stress of $\sim 5MPa$. Experimentalists closed the magnetic circuit in a mumetal $^1$ yoke separated from the sample with variable nonmagnetic gap, that makes it possible to vary the differential permeability, thus to make measurements on different permeabilities: $2.5 \times 10^3 < \mu/\mu_0 < 6 \times 10^4$. The external driving field had triangular waveform, and the experiments investigated the behavior of the strip on different driving rates over a range of $1.5 \times 10^{-4}T/s < \dot{M} < 1.5T/s$ ($\dot{M} \equiv \frac{dM}{dt}$). Data were collected only from a narrow magnetization interval (including $M = 0$) from a hysteresis half-loop, because in this region the magnetic permeability is fairly constant. BN was detected by a 50 turn coil, with width 2 mm placed around the middle of the strip. This ensured that the induced voltage is proportional to the magnetic flux rate ($\dot{\Phi} \equiv \frac{d\Phi}{dt}$). The detected voltage was amplified and statistically analyzed.

Results

Alessandro et al. computed from the measured Barkhausen pulses the amplitude probability distribution function of the induced flux rate ($\dot{\Phi}$) and the noise power spectrum ($F(\omega)$). All experimental data suggests the following statistics:

$$P_0(\dot{\Phi}) \sim \dot{\Phi}^{\tilde{c}-1} \exp(-\tilde{c}\dot{\Phi}/SM),$$

$$F(\omega) \sim 1/\omega^2,$$

where $\tilde{c}$ is a parameter, and includes the dependence of the distribution on the driving rate and magnetic permeability. The scaling of the power spectrum $F(\omega)$ is valid in the high frequency region.

From the shape of the amplitude probability distribution function $P_0(\dot{\Phi})$ (see Fig. 2.2. left) Alessandro et al. concluded that for high driving rates domain walls are moving with an average velocity $v \sim \langle \dot{\Phi} \rangle$ with weak fluctuations around this value. By the other hand for very slow driving rates $P_0(\dot{\Phi})$ diverges for $\dot{\Phi} \to 0$. This corresponds to the presence of Barkhausen bursts separated by finite time intervals. Domain wall (DW) motion becomes intermittent due to the effect of pinning centers and DW-DW interactions. Similar dependence was found on the

$^1$Mumetal is a metallic alloy with very high magnetic permeability.
value of the permeability. Both the effect of the driving rate and the permeability was included in the $\tilde{c} > 0$ parameter. $\tilde{c} = 1$ defines the boundary between the intermittent ($\tilde{c} < 1$) and the continuous ($\tilde{c} > 0$) domain wall movement. Alessandro et al. found the following dependence of $\tilde{c}$ on driving rate and permeability: $\tilde{c} \sim \dot{M}^{3/2}$ in the $\tilde{c} > 1$ region and $\tilde{c} \sim \dot{M}$ when $\tilde{c} < 1$, respectively $\tilde{c} \sim \mu^{-1/2}$, where $M$ is the magnetization and $\dot{M} = dM/dt$.

For the power spectrum (see Fig. 2.2, right) they obtained: in the high frequency region $F(\omega) \sim \omega^{-2}$, but for lower frequencies a peak is characteristic to the spectrum. The frequency at which the peak occurs scales as: $F_M \sim (\dot{M})(\mu/\mu_0)^{3/2}$.

2. G. Durin and S. Zapperi in [5, 6, 26] reports on BN measurements on several different types of materials, both polycrystalline and amorphous alloys:

(1) Fe-Si 7.8 wt.% strip (30 cm x 0.5 cm x 60 µm) annealed around 950°C to obtain grains of dimension around 25 µm;

(2) Fe-Si strip 6.5 wt% (30 cm x 0.5 cm x 45 µm) annealed for 2h at 1200°C with grains of 160 µm;

(3) the same as (2) annealed for 2h at 1050°C with grain of 35 µm;
(4) amorphous alloy $Fe_{21}Co_{64}B_{15}$ ($20\text{cm} \times 1\text{cm} \times 22\mu\text{m}$);

(5) $Fe_{64}Co_{21}B_{15}$ ($28\text{cm} \times 1\text{cm} \times 23\mu\text{m}$), a tensile stress of $\sigma \sim 100\text{MPa}$ was applied during the measurement;\(^2\)

(6) partially crystallized $Fe_{64}Co_{21}B_{15}$ ($22\text{cm} \times 1\text{cm} \times 23\mu\text{m}$) annealed for 30 min at $350^\circ\text{C}$ and then for 4h at 300$^\circ\text{C}$ under tensile stress of 500$\text{MPa}$, procedure which induces the formation of $\alpha - Fe$ crystals of about 50 nm with a crystal fraction of around 5%.

Data collecting

All the measurements were performed with the smallest available magnetic field rates of frequency $f = 3 - 5\text{mHz}$. The avalanche size-, respectively the avalanche duration distributions were plotted. Histograms show that the materials belong to two different classes characterized with different exponents.

Results

Scaling laws were obtained for jump-size- and duration distribution (see Fig. 2.3):

\[
P(S) = S^{-\tau} f(S/S_0), \tag{2.3}
\]

\[
P(T) = T^{-\alpha} g(T/T_0). \tag{2.4}
\]

For (1) Si-Fe policrystals and the partially crystallized amorphous alloy they obtained $\tau = 1.50 \pm 0.05$ and $\alpha = 2.0 \pm 0.2$; (2) for the amorphous alloys under stress the obtained exponents were $\tau = 1.27 \pm 0.03$ and $\alpha = 1.5 \pm 0.1$. Exploring the effect of the demagnetizing factor $k$ (explained later in the presentation of the infinite- and long-range domain wall model in section 3.3.3) on the cutoff yielded the following results: $S_0 \sim k^{-1/\sigma_k}$ with $\sigma_k \approx 0.78$; $T_0 \sim k^{-\Delta_k}$ with $\Delta_k \approx 0.4$.

3. Urbach, Madison and Markert (UMM in the followings) \cite{4} performed an experiment using thin foils ($100\mu\text{m}$) from 30% Fe, 45% Ni, 25% Co alloy (Perminvar) cut into ellipses with dimensions: 2 mm $\times$ 5 mm. The samples were annealed at 1000 $^\circ\text{C}$ for 1h and at 450 $^\circ\text{C}$ for 24h. Perminvar prepared through this procedure has lower susceptibility than usual soft ferromagnets (around 250), because during the annealing domain walls were strongly pinned. In the case of Barkhausen noise

\(^2\)materials (4) and (5) are highly magnetostrictive ($\lambda_s \sim 10^{-5}$)
Figure 2.3: (left) Experimental avalanche size distribution from Ref. [5]. Solid line has the slope $\tau = 1.5$, the dashed line’s slope is $\tau = 1.27$, corresponding to the two universality classes; (right) Experimental avalanche duration distribution from Ref. [5]. Solid line has the slope $\alpha = 2$, the dashed line’s slope is $\alpha = 1.5$, corresponding to the two universality classes.

studies this property is an advantage because Barkhausen events are well separated if the external field is changed slowly.

Data collecting

After completing the security operations (magnetic shielding, etc.), the Barkhausen signal was acquired from a 50 turn coil wrapped around the sample, and the amplified voltage was recorded with a 1MHz, 12 bit data acquisition board. The high speed recording was needed due to the short duration of pulses which were of the order of milliseconds. They also applied a threshold 0.5% of the maximum peak height. 66 000 avalanches over 440 cycles were collected from a small field increment. The total cycle was ranging from -5 Oe to 5 Oe and the driving had triangle wave form with a period of 75 sec. Data were taken from 4 to 4.5 Oe.

Results

Instead of voltage size distribution Urbach et al. presents the signal area distribution $P(A)$ (see Fig. 2.4). Signal area is the quantity that measures the area under a voltage peak. The reason why they considered this quantity relevant was that it corresponds directly to the number of magnetic moments reversed during a certain event. They found power-law distribution with exponential cutoff obeying:

$$P(A) \sim A^{-\alpha} \exp(-A/A_0),$$  \hspace{1cm} (2.5)
Figure 2.4: Experimental avalanche size distribution obtained by Urbach et al. [4]. Inset: autocorrelation function

with $\alpha = 1.33$ and $A_0 = 1.4 \times 10^4$.

The effect of the demagnetizing fields on the Barkhausen signal $B(t)$ is clearly shown on the autocorrelation function inset of Fig. 2.4:

$$S(\tau) = \int dt B(t)B(t + \tau).$$  \hfill (2.6)

This autocorrelation function exhibits a dip at the origin, which means that peaks repel each other at short times. From the graph one can read that the negative correlation’s time scale is around 200 msec, since the average spacing between peaks was around 10 msec. Urbach et al. find the explanation for this phenomenon in the effect of the long-range demagnetization. Details will be given in section 3.3.1, where the UMM model will be presented.

4. Spasoievic et al. [3] performed BN measurements and data analysis on a quasi-two-dimensional metal glass sample (a commercial VITROVAC 6025 X) with $4cm \times 1cm \times 0.003cm$, no mechanical stress was applied.

Data collecting

Data collection was done through a standard inductive experimental setup: the signal generator provided sinusoidal driving field with small $f = 0.03Hz$ frequency for a 20cm long solenoid. BN was recorded through a pickup coil of length 5.5cm. The specimen was placed inside the pickup coil (see Fig. 2.1.).

Results

Let the Barkhausen noise time-signal be $F(t)$. The signal duration is the time interval $T$ between the first $t_f$ and the last moment $t_l$ when the signal is above
a previously defined discrimination level (threshold or baseline). The area of the signal $A$ is defined as the area between the signal and the baseline $b_l$:

$$A = \int_{t_l}^{t_f} [F(t) - b_l] dt. \quad (2.7)$$

Physically the area of the signal is proportional to the change in the magnetization during the signal duration. Signal energy $E$ is defined to be proportional to the integrated squared signal:

$$E \sim \int_{t_l}^{t_f} [F(t) - b_l]^2 dt. \quad (2.8)$$

For all the three quantities defined above they found a power-law distribution over several decades (see Fig. 2.5):

$$P(T) \sim T^{-\alpha} \quad \alpha = 2.22 \pm 0.08,$$

$$P(A) \sim A^{-\tau} \quad \tau = 1.77 \pm 0.09,$$

$$P(E) \sim E^{-\epsilon} \quad \epsilon = 1.56 \pm 0.05. \quad (2.9)$$

The power spectrum was obtained by performing Fast Fourier Transform (FFT) on the BN signal (with $2^{13}$ data). According to the estimations $S(f) \sim 1/f^\beta$ with $\beta = 1.6 - 1.7$. This result shows that in this experiment BN turned out to be between $1/f$ and $1/f^2$ (Brownian) noises.

5. Detailed experimental measurements and results were found also in the PhD work of N. Meyer. This work investigates the relationship between microstructure and magnetic properties under sinusoidal driving of a 430Nb (17% Cr) stainless steel from Ugitech SA. This material is used for automotive actuation because it presents both a good corrosion resistance and rather soft magnetic properties. Measurements
were performed in collaboration with O. Geoffroy at the prestigious Institute L. Neél in Grenoble.

In order to avoid demagnetizing field effects, the samples have toroid shape, the section is a square of $3 \text{ mm} \times 3 \text{ mm}$, the average length of the toroid is 132 mm. The alloy was annealed at $850^\circ \text{ C}$ for 30 minutes in order to remove dislocations that are pinning points for domain walls’ (DW) movement. After heat treatment the sample was cooled down in two different ways: (1) in air at room temperature – this means that some elastic stresses developed inside; (2) with slow cooling after annealing. As a consequence no internal stresses are expected in this second sample.

Data collecting

In this case a sinusoidal driving field was applied on the samples with frequencies varying from $0.25 - 400 \text{ H} \text{ z}$ and peak induction between $0.5 - 1.5 \text{ T}$. During the standard inductive measurement process magnetic field intensity ($H[\text{A/m}]$) and corresponding induction ($B[\text{T}]$) data were collected.

The group was interested in studying the effect of the driving rate on the shape of the hysteresis loops. Since the area enclosed in the hysteresis loop is a measure of the volumic energy loss during the magnetization-demagnetization cycle, the research group investigated the conditions for lowest possible energy loss as a function of the frequency of the external driving field. Similarly, the induction-magnetization correlation was calculated as a function of driving frequency.

Results

The results of the measurements indicated that the volumic energy loss monotonically increases with increasing driving frequency in case of both the air quenched and slowly cooled samples. By the other hand the induction-magnetization (BM) correlation is different for the two samples: (1) for the air quenched material the correlation monotonically decreases with increasing frequency; (2) for the slowly cooled material the BM correlation has a slight minimum at frequencies around 100 Hz for the measurements with 0.5 T and 1 T peak induction.

As we can conclude from these experimental results different materials clearly belong to different universality classes characterized with different critical exponents. On the other hand it is a crucial factor whether the material was submitted to previous preparation (annealing, stress), because these procedures induce fundamental changes in the domain
structure and disorder type. Different experiments were performed with slightly different experimental setup. This again produces differences in the collected data, and in the analysis process. Some experimentalists [25] argue that data are reliable only in the linear region of the hysteresis loop, others [3] average data collected from the whole hysteresis. The driving rate also influences [27] the experimental data.

All these factors contribute to the widely different results obtained from specific experimental measurements, thus it would be very difficult to find a universal theoretical explanation suitable to reproduce all experimental results.
Chapter 3

Review of Theoretical Models Aimed to Explain Magnetization Phenomena in Ferromagnetic Materials

As we learned in the previous chapter a typical BN experiment yields a histogram of Barkhausen pulse duration, sizes, energies. The distributions are usually power laws over two or more decades. This scaling regime is much larger than any observed morphological feature, thus the explanation for it must involve collective motion of many domains. One expects that this explanation should also account for the cutoff in the distribution function.

There are several requirements for a model or theory that wants to explain BN [28]. A model has to explain the statistical properties of BN, such as power spectrum, avalanche size distribution, signal area and signal duration distribution, pulse shape, and explain the origin of the cutoff in the scaling regime. The theory is expected to be able to give explanation for the occurrence of scaling, to predict the values of the scaling exponents which characterize the distribution functions. It should be able to relate noise properties to the microscopic structure of the ferromagnetic material, and to explain the effect of microstructural parameters (like lattice structure, anisotropies, different types of interactions) on the noise statistics. All these requirements are hard to be fulfilled by one single theory. The situation is however not so gloomy since all experiments report the existence of scaling and power-laws for distribution functions characteristic to BN. This
is the signature of critical phenomena, where universality implies that only main symmetries and conservation laws influence the system’s behavior, and many quantitative details are irrelevant. This universal feature makes it possible that relatively simple models can capture the main properties of magnetization phenomena, and are able to reproduce (at least qualitatively) a wide number of experimental results.

Before the description of particular BN models I will briefly present the basic interactions that are governing magnetic systems.

Ferromagnetic materials can be represented as an ensemble of magnetic moments or spins localized on a lattice. The elements of this ensemble interact with each other and with an external driving magnetic field $\vec{H}$. All the macroscopic properties of a ferromagnetic material could be described by a microscopic theory including the basic interactions.

The energy of a ferromagnetic material is composed of the terms:

$$
E = E_{ex} + E_{m} + E_{an} + E_{dis},
$$

where $E_{ex}$ represents the exchange interactions, $E_{m}$ magnetostatic energy, $E_{an}$ the energy due to anisotropy and $E_{dis}$ the energy due to the disorder.

**Exchange energy** is the most important component that comes from short-range ferromagnetic coupling, and tends to align spins $\vec{s}(\vec{r}_i)$:

$$
E_{ex} = \sum_{ij} J(|\vec{r}_i - \vec{r}_j|)\vec{s}(\vec{r}_i)\vec{s}(\vec{r}_j).
$$

The $J(x)$ strength of the coupling decays rapidly and the summation is taken over all pairs of spins. Exchange is a pure quantum-mechanical effect and has no classical analog. In some models (for example in Ising type models) exchange is approximated with nearest-neighbor interaction.

**Magnetostatic energy** is due to the interactions of the spins with the external field and due to magnetic dipole-dipole interactions leading to an internal demagnetizing field $\vec{H}_{dm}$. $\vec{H}_{dm}$ can be considered constant inside the sample and proportional with the magnetization only in the case of some special geometries, such as a uniformly magnetized
ellipsoid [28]. Generally
\[ E_m = -\frac{\mu_0}{8\pi} \int d^3r \vec{M} \cdot (\vec{H} + \vec{H}_{dm}(\vec{r})). \] (3.3)

**Magnetocrystalline anisotropy and magnetoelastic energies** will yield the third term in equation (3.1). The magnetization of a ferromagnet is typically preferential: there is an easy magnetization axis (corresponding to the crystallographic axis) along which a ferromagnet is usually magnetized. This is called magnetocrystalline anisotropy. On the other hand the magnetostrictive properties of the material can lead also to a magnetoelastic energy. In the most simple case of uniaxial crystal with isotropic magnetostriction under uniaxial stress this energy contribution has the form
\[ E_{an} = \int d^3r K(\vec{M} \cdot \vec{e})^2, \] (3.4)
where \( \vec{e} \) is the easy axis. This form stands for both the uniaxial anisotropy (than \( K \) stands for the uniaxial anisotropy constant \( K_0 \)) and the uniaxial magnetostriction (than \( K \) stands for the uniaxial magnetostriction constant \( K_0 + 3/2\lambda\sigma \)).

**Disorder** in a ferromagnetic system is usually due to the presence of crystalline defects (vacancies, dislocations), or non-magnetic impurities, the grain boundaries and variations of the anisotropy axis in polycrystals. All these will give an extra contributions to the magnetic free energy of the ferromagnet, and are responsible for fluctuations and BN. The abovementioned disorders vary randomly in space, but usually they do not evolve in time. They are called thus quenched disorder.

Disorder determines crucially hysteresis properties, especially in the rate-independent limit. This limit is characterized by: (i) athermal character and (ii) small driving rate. Ideally we would also need: zero-temperature and quasistatic driving [13].

Energetically the non-magnetic impurities cause random fluctuations in the exchange coupling (or in the dipolar coupling), this is called random-bond type disorder, and is characteristic mainly to amorphous materials. In polycrystalline and magnetostrictive samples the random distribution of anisotropy axes and internal stresses play the role of disorder. This is a random anisotropy-type disorder. Another way of dealing with disorder theoretically is to take it as a random field. Despite the fact that random field is only a property characteristic for antiferromagnets, it is a useful theoretical tool even in modeling disorder in ferromagnetic materials (see section 3.2.1).
Depending on which types of interactions and in what way they are taken into account, what type of approximations are made in order to simplify the treatment of the problem, the models for Barkhausen noise can be divided in several different classes.

### 3.1 The ABBM Model

As we learned before the most important ingredient necessary for the occurrence of Barkhausen noise is the presence of disorder. Thus a simple possibility is to neglect any other detail in the description of the phenomenon and to take into account only a uniform magnetostatic term modulated by a random value which would model the influence of all defects and impurities. This was the starting idea of Alessandro et al. [24], who constructed the first model that gave a quite accurate description of the BN statistics.

In their paper Alessandro et al. [24] theoretically investigated the stochastic motion of a domain wall (DW) in a randomly perturbed medium through the theory of stationary Markov processes. Williams, Shockley and Kittel [29] have shown in 1950 that DW motion is governed by eddy current damping which results that the DW velocity \( v \) depends linearly on the internal magnetic field \( H \), and its properties come from the macroscopic magnetization law of the system:

\[
kv = H - H_c, \tag{3.5}
\]

where \( H_c \) is the coercive field experienced by the moving DW. \( H_c \) is a random function of DW position and represents the stochastic disturbance caused by microscopic DW interactions. This is responsible for the intermittent character of the BN. Eq. (3.5.) plays the role of a Langevin equation. Alessandro et al. in Ref. [24] solve this Langevin equation and the associated Fokker-Planck equation in order to obtain results on the statistical properties of BN.

The original paper where the ABBM model is introduced and studied [24] emphasizes that different magnetization processes take place when different regions of the hysteresis loop is considered. Thus, they affirm that averages taken over the entire hysteresis loop "are not easy amenable to a clear physical interpretation". By the other hand BN is a non-stationary process, though it is approximated with a stationary Markov process.

The model considers one single 180 degree domain wall moving in a metallic slab. DW flexibility is neglected, thus DW motion is described by its velocity, or the equivalent
magnetic flux rate. The dynamics of the DW is governed by an equation similar to (3.5.):

$$\sigma G \dot{\Phi} = H - H_c.$$  \hspace{1cm} (3.6)

In this equation $H_c$ is a random function of the induced magnetic flux ($\Phi$), and it obeys a Langevin equation. This term gives the stochastic character for the system’s behavior originating from the DW interactions with lattice defects, non-magnetic impurities and other random perturbations. By the other hand the internal magnetic field $H$ contains the applied field and the magnetostatic effects also. In the framework of this model the magnetostatic field’s change is taken to be proportional with the magnetic flux rate ($\dot{\Phi}$). This assumption could be made with the constraint that only that part of the hysteresis loop is studied where the differential permeability is constant and the applied external field is also increasing by constant rate.

The statistical properties of BN were calculated from approximate analytical solutions of the Fokker-Planck equation associated with the Langevin model and computer simulations of DW motion were performed.

The calculations predicted for the behavior of the amplitude probability distribution ($P_0(\dot{\Phi})$) of the magnetization flux rate $\dot{\Phi}$ the following law:

$$P_0(\dot{\Phi}) \propto \dot{\Phi}^{-\tilde{c}} \exp \left( \frac{-\tilde{c}}{\langle \Phi \rangle} \right),$$  \hspace{1cm} (3.7)

with a $\tilde{c} > 0$ parameter.

A result in the form (3.7) for the amplitude probability distribution is obtained both in the low and high DW velocity limit, fact which implies that this type of amplitude distribution is a basic and characteristic feature of BN, thus it is expected to be observable also in experiments. The results obtained indicated scaling properties in the intermittent behavior of BN at low magnetization rates.

Analytical expressions for the power spectrum of the noise were also derived. In the high frequency region they obtained a Brownian type noise with $F(\omega) \propto \omega^{-2}$. At low frequencies the power spectrum has a more complicated shape: it attains a maximum at the angular frequency $\omega_M = (\tau \tau_c)^{-1/2}$ ($\tau$ is a time constant that governs the decay of the magnetostatic field and $\tau_c$ governs the decay of local coercive field correlations), and decays to zero as $F(\omega) \sim \omega^{-2}$ when $\omega \to 0$. In conclusion the low frequency power spectrum of the ABBM model indicates white noise for BN, and the maximum appears due to the interplay of the magnetostatic effects and the finite correlation length of the
coercive field fluctuations.

The ABBM model was developed for one single moving DW. The theory was extended to the case where many domain walls participate in the magnetization process. It was shown that the ABBM model holds even for many participating domain walls when $\dot{\Phi}$ is the total magnetic flux rate produced by the many moving domain walls.

### 3.2 Spin Models

Due to the fact that many, apparently very different, physical systems far from equilibrium show the characteristic hysteretic behavior, it might be possible to construct very simple models which reproduce successfully hysteresis, avalanche-like processes and their statistical properties. Such models are expected to incorporate only the most relevant information about a system, like symmetry properties, dimension and basic interactions.

It has been observed that the characteristics of the hysteresis loops are influenced by the type and amount of disorder [18] which is inevitably present in a real physical system and modifies the free energy landscape. The magnetic material becomes magnetically softer when the amount of disorder is increased. The shape of the hysteresis loop changes from sharp (this correspond to small amount of disorder) to smooth (corresponding to large amount of disorder) and the distribution of sizes and durations of Barkhausen signals is also modified. This type of transition can be interpreted as a disorder-induced first order phase transition. There is a specific amount of disorder at which the transition from sharp to smooth magnetization reversal becomes continuous. At this point avalanche size -, avalanche area -, avalanche duration distributions become critical exhibiting power-law behavior.

Based on these ideas, different versions of lattice spin models with disorder have been proposed as simple models that aim to incorporate the essential physics of disordered systems: random field- (RFIM), random bond- (RBIM), random anisotropy Ising models (RAIM).

#### 3.2.1 Random Field Ising Model

Zero-temperature random-field Ising model (RFIM) was introduced by Sethna et al. in Ref. [14]. They studied the dynamics of the disorder-driven phase transitions and related critical phenomena. The starting point was the classical model for first-order phase transition: Ising model in an external field $H$. Sethna et al. in Ref. [14] added
to the idealized picture a disorder, which is a random field $h_i$ at each site of the Ising model. The new model is athermal ($T = 0$, hence the name zero-temperature). This zero temperature model is applicable to many experimental systems where the elementary domains have large enough barriers in their way to flip so that thermal effects can be neglected.

**Components of the RFIM**

The RFIM describes an oversimplified 3D ferromagnetic system as a cubic grid of magnetic "spins" $s_i$ with $s_i = \pm 1$ (pointing up or down). Spins can represent a domain or a mesoscopic part in the material. Between neighboring domains there is a ferromagnetic coupling, with coupling constant $J$. Usually it is assumed $J = 1$ for the sake of simplicity, and in this way all the energies are in $J$ units. External driving magnetic field is applied $H(t)$. Disorder (impurity, crystalline defect) is taken into account through a random field $h_i$. This acts on each domain (spin), and takes random values according to normal distribution with standard deviation $R$ [see eq.(3.8)]. This will be the most important parameter of the model, and is called the amount of disorder.

$$P(h) = \exp(-h^2/2R^2)/\sqrt{2\pi R}. \quad (3.8)$$

Depending on whether only nearest-neighbor coupling or global coupling is taken into account, and what other types of interactions are considered for the dynamics of the system, one gets different variations of the zero-temperature RFIM. The most general Hamiltonian of the RFIM is:

$$\mathcal{H} = -J \sum_{ij} s_i s_j - \sum_i (H(t) + h_i) s_i + \sum_i \frac{J_{\text{inf}}}{N} s_i - \sum_{ij} J_{\text{dipole}} \frac{3\cos(\theta_{ij}) - 1}{r_{ij}^3} s_i s_j. \quad (3.9)$$

In the initial state of the system all the spins point down ($s_i = -1$ for all $i$). $s_i$ can flip whenever the local external field changes sign. There are two possibilities for a spin to flip:

1. $H(t)$ becomes sufficiently large and a new avalanche will born. This mechanism is called ”avalanche nucleation”.

2. The neighboring spin flips and due to the existing coupling this causes the flip of the spin in case. This situation is called the propagation of an existing avalanche.

According to the zero-temperature RFIM the system reaches its critical state for a
critical value $R_c$ of the disorder. In this case avalanches of all sizes occur and their
distribution follows power law over many decades of magnitude. In RFIM the disorder
is tunable parameter and varying this will change the ratio between disorder effects and
spin coupling. If the coupling between grains is weak compared to disorder ($J \ll R$), the
grains will flip independently, which leads to small avalanches. If the coupling is too strong
($J \gg R$), a flipping grain will cause its neighbors to flip, the newly flipped neighbors will
trigger again other flips, and in this way the process will lead to one large avalanche that
can span the whole system. In a finite system this kind of avalanche that spans almost
the whole system (or at least a finite fraction of it) is called ”spanning avalanche”, and
it is equivalent to an ”infinite avalanche” in the thermodynamic limit. A wide range of
avalanche sizes is observable at the critical value of the disorder. In such case avalanches
of all sizes ranging over several decades of magnitude occur during the magnetization
process.

Whenever an avalanche arises and propagates, it rearranges the energetic landscape
of the system. The system shifts from one metastable state to another. Small avalanches
rearrange only few spins, but huge avalanches can span over the whole system.

**Mean-field Solution and Computer simulations** The first approximated solution
was done by Sethna et al. with mean-field theory [14] on the simple Hamiltonian of the
form:

$$H = -J \sum_{ij} s_i s_j - \sum_i (H(t) + h_i) s_i .$$

In this equation every spin is coupled to all other $N$ spins with coupling strength $J/N$.
The effective field acting on one site is $J M + h_i + H$, where $M = \sum_i s_i / N$ is the average
magnetization. All sites having $h_i < -J M - H$ will point down, thus the magnetization
is given by:

$$M(H) = 1 - 2 \int_{-\infty}^{-JM(H)-H} \rho(h) dh .$$

In papers [14, 30, 31] Sethna et al. give comparative results of this model obtained by
mean-field calculations and simulations on three- to five-dimensional systems. I summa-
ris below their most important results.

1. The simulated hysteresis loops show return-point memory. This means that the
system will return exactly to the same state whenever it is under the same value of
the external field.

2. A specific "phase diagram" can be plotted in the $M(H)$ plane for different $R$ values of the disorder (see Fig. 3.1). There is a critical strength $R_c$ of the disorder above which value an infinite avalanche never occurs. For weaker disorder there is a certain value of the external field $H_c^u(R)$ (notation from [14]) at which an infinite avalanche first occurs. This transition at $H_c^u(R)$ is abrupt for $R < R_c$, but as the disorder approaches the critical field $H_c^u(R_c)$ at $R_c$, the transition will be continuous. At this point the magnetization $M(H)$ has power-law singularity, and avalanches of all sizes are present. In the neighborhood of this $(R_c, H_c^u(R_c))$ in the $(R, H)$ plane there are diverging correlation lengths and critical behavior.

3. Both mean-field calculations and simulations yield power-law with cutoff for the avalanche-size distribution.

4. Mean-field calculations performed on the RFIM yield for the critical disorder in 3D $R_c = 2.16$. It is important to note that the critical disorder is not universal.

5. The RFIM exhibits critical scaling not only at the exact value of the critical disorder, but in a wide regime around this point. The "distance" of a particular realization of the disorder from the critical value is measured by the reduced disorder, defined as: $r = \frac{R - R_c}{R}$. In Ref. [30] the authors report that even 50% away from the critical point there are two decades of scaling yet in the avalanche size distribution.
6. At the critical point the expected behavior of the system would yield the following power laws:

- The distribution of avalanche sizes measured at a field $H$, or in a small range of fields centered around $H$ at the critical disorder $R_c$ scales as:

$$D(S, H) \sim S^\tau,$$

(3.12)

where we denoted by $S$ the avalanche size.

- The integrated avalanche-size distribution is the size distribution of all the avalanches that occur in one branch of the hysteresis loop (for $H$ from $-\infty$ to $\infty$). This distribution function scales as (see Fig. 3.2.):

$$D_{int}(S, R_c) \sim S^{-(\tau+\sigma\beta\delta)}.$$

(3.13)

- The scaling breaks down after two to several decades of scaling at an avalanche size:

$$S_{\text{cut off}} \sim (R - R_c)^{-1/\sigma}.$$

(3.14)

- The magnetization scales with a power law $(M - M_c) \sim (H - H_c)^\delta$ at the critical disorder $R_c$, and the jump in the magnetization scales as $\Delta M \sim (R - R_c)^\beta$ as one varies the disorder below $R_c$.

- The mean-field values for the exponents introduced above are: $\tau = 3/2$, $\sigma = 1/2$, $\beta = 1/2$, $\delta = 3$.

- The exponent $\nu$ characterizes the divergence of the correlation length $\xi$ at the critical field $H_c$ and near the critical disorder $R_c$: $\xi \sim |R - R_c|^{-\nu}$.

- The scaling relation between the time $t$ it takes an avalanche to occur and the linear size $\xi$ of the avalanche defines the critical exponent $z$, known as the dynamical critical exponent. The time needed for a spin to "feel" the flip of another spin at a distance $\xi$ scales as $t \sim \xi^z$, and the time scales with the avalanche size as: $t \sim S^{\sigma\nu z}$.

7. This kind of mean-field approach yields two unphysical artifacts:

- There is no hysteresis apart from the infinite avalanche.
The transition from abrupt jump in magnetization (low value of disorder) to the smooth hysteresis curve (high value of disorder) when $R < R_c$ is continuous in mean-field theory, since numerically simulations indicate that the transition is discontinuous.

Dahmen and Sethna argued [18] that due to universality the system is expected to behave in a similar manner if different lattice, different disorder distribution or more realistic random anisotropy is introduced. For this reason very simple models, such as RFIM (or the spring-block model and spin Hamiltonian model which will be later introduced) can reproduce the main characteristics of crackling noises (for ex. BN). These models, however, are not suitable to explain and quantitatively reproduce real experiments.

Critical behavior and the values of the critical exponents are not the only predictions of the RFIM. In Ref. [30] the authors report results after performing scaling collapses, which predict the shape of the curves and the way how they cut off. For the integrated avalanche-size distribution the authors in Ref. [30] obtained:

$$D_{int}(S, R) \sim S^{-(\gamma+\sigma\delta)} \cdot D_{int}(S^\sigma r),$$

as $r = \frac{R-R_c}{R} \rightarrow 0$.

The scaling function $D$ is a constant for small arguments and dies away exponentially for large arguments. The main point is that the whole $D(x)$ function is just as universal as the critical exponents.
Performing data collapse means plotting
\[ D_{\text{int}}(S^\sigma \cdot \frac{R - R_c}{R}) \sim S^{\tau + \sigma \beta} \cdot D_{\text{int}}(S, R). \] (3.16)

Data taken at different \( R \) will all collapse onto the same curve (see inset of Fig. 3.2). This was discovered by Widom: data for systems near criticality can be collapsed onto one another. According to this theory any other system belonging to the same universality class will also rescale in the same way.

Applying the scaling law backwards, one can make predictions for the avalanche size distribution for different values of the \( R \) disorder. This has been done and it fits well the simulated data (see thin lines on Fig. 3.2).

In the table below I summarize for comparison the main exponents obtained in the RFIM from simulations in three dimensions and the theoretical results obtained with mean-field calculations:

<table>
<thead>
<tr>
<th>( MF )</th>
<th>( 3D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau + \sigma \beta \delta )</td>
<td>9/4</td>
</tr>
<tr>
<td>( \tau )</td>
<td>3/2</td>
</tr>
<tr>
<td>( 1/\sigma )</td>
<td>2</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1/2</td>
</tr>
<tr>
<td>( \sigma \nu \zeta )</td>
<td>1/2</td>
</tr>
</tbody>
</table>

(3.17)

3.2.2 Random Bond Ising Model

In paper [15] Vives and Planes introduced another spin-model. Unlike in the RFIM, the disorder is not introduced as a static random field, but on the spin-spin interactions through random coupling constants.

Components of the RBIM

This is a standard spin-glass model, defined on a 2D \( L \times L \) square lattice, with an \( s_i = \pm 1 \) spin on each site. The Hamiltonian governing the dynamics of the system is:

\[ \mathcal{H} = \sum_{<ij>} J_{ij} s_i s_j - H \sum_{i=1}^{L^2} s_i, \] (3.18)

where \( J_{ij} \) are randomly distributed coupling constants (hence the name of the model) following Gaussian distribution with mean \( J = \sum_{ij} J_{ij} / 2N = -1 \) and standard deviation \( \sigma \) (sum in the first term refers to nearest-neighbor pairs). For this Hamiltonian the mean-
field solution ought to be exact, and the results should not depend on the particular choice of the distribution for $J_{ij}$, they depend only on $\mathcal{F}$ and $\sigma$.

**Dynamics** of the model is governed by deterministic rules corresponding to an energy relaxation process. Saturation for $H \gg 0$ is $s_i = +1$ for all spins, and $s_i = -1$ for $H \ll 0$. Energy change caused by an individual spin flip: $\Delta \mathcal{H}_i = -2s_i(H_i - H)$, where $H_i = \sum_j J_{ij} s_j$ the internal field acting on $s_i$. Dynamics proceeds in similar manner like in RFIM, with two different mechanisms adoptable. (1) Synchronous dynamics: all the unstable spins flip simultaneously, and then all $H_i$ are updated. (2) Sequential dynamics: the most unstable spin flips first, the neighboring $H_i$ are updated afterwards. The two mechanisms lead to the same scaling properties and exponents, but the advantage of the first mechanism is that it enables to distinguish between the avalanche size (= the number of spins flipped during the avalanche) and the avalanche duration (= number of times that the algorithm is repeated until equilibrium is reached). Vives and Planes performed simulations with system sizes ranging from 10 to 100, and took averages for up to 5000 equivalent systems. Their results were reported in Ref. [15].

**Hysteresis loops** obtained for different amounts of disorder $\sigma$ are shown on Fig. 3.3 (left). It is clearly observable that there has to be a critical disorder $\sigma_c$, at which the discontinuous magnetization reversal changes to continuous. Below $\sigma_c$ the hysteresis shows big jumps, while for high disorders the loop becomes smoother. At $\sigma_c$ the macroscopic jump in magnetization shrinks to a point, in the vicinity of this point the slope of the curve approaches infinity. This is an evidence of a disorder-induced continuous phase-transition.

Fig. 3.4. shows an inner loop (partial cycle), and we can read from it that, contrary to the RFIM, in the case of the RBIM no return point memory is detectable. The explanation is in the fact that reverse spin flip is allowed to occur in RBIM inside an avalanche due to the presence of antiferromagnetic bonds in the system (although their fraction is usually small, around 10%). This destroys the partial ordering in the system, which is a basic requirement for return point memory to occur.

They have found that criticality is valid in a quite wide range of disorders around the critical value: at least between $\tilde{\sigma} = -1.5$ and $\tilde{\sigma} > 5$, where $\tilde{\sigma} = (\sigma - \sigma_c(L))/\sigma_c(L)$.

Three different methods were used in order to locate the critical point:

1. Defining $<m_0>/L^2$ (the average size of the largest avalanche) as the order parameter, the inflexion point of the $<m_0>/L^2$ curve as a function of the disorder $\sigma$ indicates the location of the critical point (see Fig. 3.5. (left) for different system sizes and for different disorders). Due to the fact that the systems are finite,
Figure 3.3: Hysteresis loops (left) and avalanche size distribution (right) obtained from the RBIM for system size $L = 40$ and for $\sigma = 0.45$ (a), $\sigma = 0.55$ (b), $\sigma = 1.5$ (c)(after [15])
Figure 3.4: Full and partial hysteresis cycle (after [15]) showing the absence of the return point memory effect.

Figure 3.5: (left) Behavior of the $\langle m_0 \rangle / L^2$ mean size of the largest avalanche for different system sizes; (right) Scaling of the average duration of the longest avalanche $\langle t_0 \rangle / L$ for different system sizes (after [15]).
finite-size scaling was performed for different system sizes:

\[
\langle m_0 \rangle / L^2 \sim L^\beta F_{(m_0)} (L^{1/\nu} \tilde{\sigma}_L),
\]

and have found \( \beta = 0.065 \pm 0.1 \) Estimations were done for \( \sigma_c \) in the limit \( L \to \infty \) and yielded \( \sigma_c = 0.44 \pm 0.1 \) and \( \nu = 1.4 \).

2. Defining \( < t_0 > / L \) (the average duration of the largest avalanche) as the order parameter, the maximum point of the \( < t_0 > / L \) curve as a function of the disorder \( \sigma \) indicates again the location of the critical point.(see Fig. 3.5. (right) for different system sizes and for different disorders). According to the scaling hypothesis the average duration of the longest avalanches scales as:

\[
\langle t_0 \rangle \sim L^z F_{(t_0)} (L^{1/\nu} \tilde{\sigma}_L).
\]

RBIM gives \( z = 1.2 \pm 0.1 \) and \( \nu = 1.4 \pm 0.1 \). These values for the exponents give good overlap for different system sizes using the (3.20) scaling function.

3. After performing the finite-size scaling they obtained the following distribution function for the avalanche sizes:

\[
p_\sigma (m) \sim exp[-K(m/\xi^2)^\tau] \cdot m^{-\tau},
\]

where \( K \) is a constant that changes sign at the transition point. At the transition point the correlation length diverges \( (\xi \to \infty) \), and the distribution (3.21) is a pure power-law (Fig. 3.3.b.). Out of criticality two cases are possible: (i) supercritical behavior \( (\sigma < \sigma_c) \) and we have then a positive exponential prefactor \( (K < 0) \); the distribution function decays slower than power-law, see Fig. 3.3.a.; (ii) subcritical region \( (\sigma > \sigma_c) \) with negative exponential prefactor \( (K > 0) \), and the distribution function decays faster than power-law, as can be observed on Fig. 3.3.c.

Critical exponents obtained by the three different methods were all in agreement with each other.

An important feature that was pointed out in the discussion of paper [15] is the connection between the phase transition in the RBIM and the 2D percolation problem. They found that the number of sites (normalized to the system size) at the critical state in the percolation problem is 0.5928, while the size of the critical avalanche in the RBIM
is \(< m_0 > / L^2 = 0.604 \pm 0.01\), values being very close to each other. This means that the phase transition in the RBIM occurs for that value of the disorder at which the largest avalanche percolates.

### 3.2.3 Random Anisotropy Ising Model

In magnetic materials a relevant source of disorder is the randomness of the magnetocrystalline anisotropy. RAIM is introduced in Ref. [13] to study the zero-temperature hysteresis and avalanche properties in systems with such disorder. This is a random uniaxial model, and is considered to be suitable for describing magnetic properties of amorphous alloys. In addition it has been argued on symmetry grounds [18] that RAIM should belong to the same universality class as the RFIM.

#### Components of the RAIM

RAIM considers a 3D cubic lattice with spins \(s_i\) and local anisotropy axes \(\vec{n}_i\) defined for each site and described by a pair of spherical polar coordinates \((\theta_i, \phi_i)\). The anisotropy axis directions are random and distributed according to a generic normalized probability density distribution function \(f(\theta, \phi)\). The Hamiltonian of the system is:

\[
H = -\sum_{<ij>} J_{ij} s_i s_j - H \sum_{i} s_i \cos \theta_i,
\]

where the first term is a random bond term with \(J_{ij} = J\vec{n}_i \cdot \vec{n}_j\) \((J = 1\) for simplicity) and the summation is over nearest-neighbor pairs. The second term means the random coupling with the external magnetic field. This latter describes a quenched disorder, since the values \(\theta_i\) does not change in time.

Two different distributions of anisotropy axes have been studied. In both cases the amount of disorder in the system is controlled by a given parameter.

**In version A**

\[
f(\theta, \phi) = \frac{1}{2\pi(1 - \cos \theta_0)} h[\theta_0 - \theta],
\]

and the amount of disorder is controlled by parameter \(\theta_0\), and \(h[x]\) is the step function.

**In version B**

\[
f(\theta, \phi) = (1 - \epsilon) \frac{1}{2\pi} + \epsilon \frac{3}{2\pi} \cos^2 \theta,
\]

where the disorder is controlled by \(\epsilon\).

An important first result is that the systems critical behavior is qualitatively not influenced by the particular choice of the distribution. This observation is important
because it is an indicator of the universality: the scaling properties do not depend on the
details of the system, but only on the dimensionality and the main symmetry properties.
Figure 3.7 proves this universal character showing that the distribution of the avalanche
sizes is qualitatively the same for the two different probability density functions.

The dynamics of the model is the so-called field-driven nucleation [14], which treats do-
main nucleation and growth simultaneously. Under slow changes of the external magnetic
field, the system follows deterministic dynamics corresponding to local energy relaxation,
thus the system will evolve through metastable states.

The starting point is the saturation state. Changing the external driving field, the
energy change associated with an independent spin flip is:

\[ (\Delta H)_i = 2F_is_i \]  
\[ F_i = \sum_{<ij>} J_{ij}s_j + H\cos\theta_i. \]  

Metastable state is for which \( \Delta H_i > 0 \) for \( \forall i \). When for a certain value of \( H \) one site
becomes unstable, that spin flips and \( H \) remains constant. If this flip destabilizes neigh-
boring spins, these spins will flip simultaneously. This is the beginning of an avalanche. \( H \)
is unchanged until all spins become stable again. This fulfills the condition of rate-
independent hysteresis.

Simulations were performed with both variations of the model, results were reported
and compared in Ref. [13].

The results on hysteresis shapes, avalanche duration \( t \) (= number of avalanche steps)
and avalanche size \( S \) (= total number of spins flipped during an avalanche) will be pre-
sented in the followings.

**Hysteresis loops** have qualitatively the same symmetrical shape for both variations
of the RAIM model. It is clear from the graphs (see Fig. 3.6) that there must be a critical
value of the disorder, below which infinite avalanches occur (an avalanche is called infinite
in a finite system when its size is a finite fraction of the system size), above it only very
small avalanches are observable.

Vives and Planes in Ref. [13] verified whether the return point memory property is
fulfilled. They have found, studying inner loops, that this condition is not fulfilled because
RAIM allows reverse spin flips during an avalanche (due to the continuous distribution of
anisotropy axes), although these occurrences represent less than 5% of the total number
of flips.
CHAPTER 3. REVIEW OF THEORETICAL MODELS

Figure 3.6: Hysteresis loops obtained with the RAIM using distribution A (after Ref. [13])

- $\theta_0=1.0$
- $\theta_0=1.3$
- $\theta_0=\pi/2$

Figure 3.7: Avalanche size distributions for version A and B of RAIM (after Ref. [13]) for different values of the disorder
Avalanche size distribution functions were plotted for different disorder values averaged over 300 different half-loops (see Fig. 3.7.) For small values of the disorder a very large avalanche, and a few small ones can be observed. The power-law scaling is not clear. This is the so-called "supercritical" region. For large values of the disorder the hysteresis consists only of small avalanches, this is called "subcriticality". The critical disorder was found to be $\theta_0^c = 1.44 \pm 0.01$ (model A) associated with power-law avalanche size distribution characterized by a critical exponent $\tau = 2.06 \pm 0.05$. For model B it was found $\epsilon^c \sim 0.2$ with power-law exponent $\tau = 2.10 \pm 0.05$.

In a similar manner like in the case of the RBIM, three independent methods for locating the critical point were used:

1. Detect the inflexion point of the $<s_{max}/L^3>$ versus the disorder (see Fig. 3.8 upper graph), and perform finite-size scaling for different system sizes:

$$S_{max}(x, L)/L^3 \sim L^{-\beta/\nu} F_s(xL^{1/\nu}), \quad (3.27)$$

where $x$ is the reduced disorder, $x = [\theta_0 - \theta_0^c(L)]/\theta_0^c(L)$.

2. Locate the maximum of the $<t_{max}/L>$ versus the disorder (see Fig. 3.8 lower graph), and perform finite-size scaling for different system sizes:

$$T_{max}(x, L) \sim L^{z/\nu} F_T(xL^{1/\nu}). \quad (3.28)$$
For the critical exponents RAIM gives: $\nu = 1.0 \pm 0.1$, $\beta = 0.1 \pm 0.1$ and $z = 1.6 \pm 0.1$.

3. Analyzing the avalanche size distributions obtained for the whole hysteresis cycles for different values of the disorder. The distributions (plotted on Fig. 3.7) are well fitted with power-law with exponential cut-off:

$$p(s) \sim s^{-\tau} \cdot e^{-\lambda s}. \quad (3.29)$$

For small amount of disorder the correction exponent $\lambda < 0$, and the distribution function decays slower than power law. This is the supercritical region. In the subcritical region, where the disorder is larger than the critical value, $\lambda > 0$, the distribution decays faster than power-law. At the critical value of the disorder $\lambda = 0$, the distribution of the avalanche sizes is power-law.

### 3.3 Domain Wall Models

Models using domain wall dynamics usually have their origin in a widely applied general model which describes the dynamics of interfaces in disordered systems. This kind of dynamics is implemented in the study of fluid invasion in porous media, critical currents in superconductors, fracture propagation, domain wall motion of ferromagnets, and many other nonequilibrium growth phenomena.

These models usually include an interface elastic energy, random quenched disorder and an external driving force. Thermal fluctuations are usually neglected on long length scales. The general dynamics is the following: until the external force is weak the interface is pinned by the impurities; as the driving force reaches a critical value, the interface begins to move. The critical value of the force depends on the amount of disorder in the particular system, but near the depinning threshold the dynamics shows scaling behavior with universal exponents [4].

Experimental results prove that scaling and universal behavior over several decades of magnitude can be obtained without tuning any parameter. One of the main purposes of the Barkhausen-noise-models is to account for this very basic property.

#### 3.3.1 The UMM Model

This model, called UMM, was developed by Urbach, Madison and Markert, and presented in Ref. [4]. The purpose of the UMM model was to demonstrate that the sta-
tistical properties of BN are strongly affected by long-range demagnetizing fields. They included this interaction in a simple interface depinning model, and showed that the model’s behavior is completely different depending on the range of demagnetizing effect: an infinite-range demagnetizing field results in self-organized criticality, contrary to only local demagnetization effect.

The UMM model is based on several basic properties of ferromagnetic materials:

- The local field applied to a domain wall includes both the external field and the magnetic fields from the rest of the sample.
- Ferromagnetic susceptibility is much larger than 1, thus the demagnetizing field is almost equal, but opposite to the applied field.
- After an interface is depinned, the spins behind it decrease the net field felt by the interface (i.e. increase the demagnetizing field) until the interface is stopped. This interface dynamics holds the system always near the depinning point.

In order to prove the importance of the long-range demagnetization effects, Urbach et al. have built up a simple domain wall motion model. They describe an interface (=domain wall) by its height \( h(x, t) \), where \( x \) is a \((d-1)\) dimensional vector. In two dimensions the force on each element of the string is given by:

\[
f_i = u(h_i, i) + k(h_{i+1} + h_{i-1} - 2h_i) + H_i. \tag{3.30}
\]

The first term is the random on-site force following Gaussian distribution with mean zero. The second term is the elastic coupling between nearest-neighbor elements (with periodic boundary conditions). The third term is the driving force. Depending on the form of this latter term, there are three types of models. (1) \( H_i \) is simply the externally applied field. This version is called AF (Applied Field) model. The AF model yields simple depinning behavior at a critical value of the external field.

More interesting results can be obtained if demagnetizing effects are included, and \( H_i \) means the internal field. (2) When infinite-range demagnetizing field is present, we speak about global IF (Internal Field) model: \( H_i = H - \eta M \), where \( M = \frac{1}{L} \sum_{i=1}^{L} h_i \) for a lattice width \( L \). (3) Local IF model: \( H_i = H - h_i \). All these models were explored by simulation. Parameters were chosen to have \( H - \eta M \gg 1 \).

Results
1. **AF model**: For the growth within 4% of the critical field in a 2D system of $L = 250$ (thus the interface is a line) Urbach et al. have found power-law for the avalanche size (area) distribution over 5 orders of magnitude: $P(A) \sim A^{-\tau}$, with $\tau = 1.00 \pm 0.01$. For a 2D interface (i.e. 3D system) of $40 \times 40$ they found $\tau = 1.17 \pm 0.05$.

2. **Global IF model**: 1D interface of size 500 was simulated. Probability distribution function of avalanche sizes (area) has power-law form, slope $\tau = 1.00 \pm 0.01$ cut off governed by finite size effects. (See Fig. 3.9.) Varying parameter $k$ from 20 up to 1000, Ref. [4] reports that the value for $k$ had no detectable effect on the slope. Simulations with $40 \times 40$ 2D interface yielded an exponent of $\tau = 1.13 \pm 0.02$. These values indicate that the global IF and AF models produce the same exponents when AF model is tuned to criticality. Numerically they found that the net internal field $H - \eta M$ fluctuates around the value of (slightly below) $H_c$ from the AF model. (Parameters are the same in both cases.)

3. **Local IF model** exhibits different behavior. Avalanche size distribution scaling is only approximate, with slope slightly smaller than 1, and the statistics does not improve for larger system sizes.

4. Comparing the autocorrelation function for the global and local IF models (see inset of Fig. 3.9.) a relevant difference can be observed. The global model produces a significant negative correlation at short times, similar to the one observed experimentally (see the corresponding experimental results in section 2.1.). By contrast, the local model does not show any correlation. The presence of negative correlation in the global model, and its qualitative agreement with the experiment proves the importance of the long-range demagnetizing interaction. An explanation of this...
correlation can be given. The peak repulsion is a consequence of the demagnetizing fields: after a part of a domain wall moves as a response to an increasing field, the effective field on the domain wall is decreased. This way the probability for subsequent events to occur is reduced until the applied field is increased enough to bring the total field back to the critical depinning value.

### 3.3.2 Narayan’s Model

Onuttom Narayan’s model [32] is seemingly very similar to the UMM model. The difference is that Narayan does not consider any approximation for the dipolar interaction. He’s aim is to verify whether the exact treatment of dipolar interaction leads to self-similar behavior or not. The equation of motion for the domain wall dynamics is similar to the UMM (3.30) equation:

\[
\Gamma \partial_t h(x, t) = k \partial_x^2 h(x, t) + u[h(x, t), x] + H + \int dx' K(x, x') h(x', t),
\]

where \( \Gamma \) is a constant characterizing the amount of dissipation, the first term on the right hand-side is the surface tension of the domain wall, second term is the pinning force due to impurities and \( H \) is the applied magnetic field. The last term results from the expansion to the second order in \( h(x, t) \) of the dipolar interaction energy. The main feature of the restoring force obtained from the dipolar interactions is that it is small for large system sizes, which means that the cutoff to the self-similar behavior diverges with system size. Consequently the only cutoff is due to standard finite size effects.

Deriving an expression for the power spectrum from the domain wall velocity-velocity correlation function yielded power-law form with exponent around 1.5 for a 2D interface [32]. Narayan claims that numerically this exponent will change in different dimensions, for example in 1D it should be zero.

### 3.3.3 Infinite- and Long Range Domain Wall Model

Stefano Zapperi et. al. [23, 26, 28, 33, 34] approaches the BN problem studying the motion of a flexible domain wall (DW) driven through disordered medium in an anisotropic material magnetized along the easy axis (\( x \) axis) with 180° domain walls separating regions of opposite magnetization. Disorder pins the free movement of the DW which is driven by the external driving magnetic field. This model assumes localized disorder (or only short-range correlated disorder) and a flexible DW (see Fig. 3.10).
The flexible domain wall is described by its position $h(\vec{r}, t)$, a function of space and time. The equation of motion governing the dynamics of the DW is:

$$\Gamma \frac{\partial h(\vec{r}, t)}{\partial t} = -\frac{\delta E(\{h(\vec{r}, t)\})}{\delta h(\vec{r}, t)},$$  

(3.32)

where $\Gamma$ is the effective viscosity, $E(\{h(\vec{r}, t)\})$ is the total energy functional. The total energy is composed of different contributions as presented in the introduction of this chapter. This model takes into account the different energetic contributions in the following way [23]:

- Magnetostatic energy and the contribution from the demagnetizing fields:

$$E_m = -2\mu_0 H M_s \int d^2 r h(\vec{r}, t),$$  

(3.33)

$$E_{dm} = \frac{2\mu_0 N M_s^2}{V} \left( \int d^2 r h(\vec{r}, t) \right)^2,$$  

(3.34)

with: $M_s$ - saturation magnetization per unit volume; $N$ - the demagnetizing factor which takes into account also the sample geometry. This term was also included in the UMM model.

- Dipolar interactions are treated by introducing a "magnetic charge" density for a domain wall separating two regions of different magnetization:

$$\sigma = (\vec{M}_1 - \vec{M}_2) \cdot \vec{n}.$$  

(3.35)
In expression (3.35) \( \vec{n} \) is a normal to the surface. The energy associated with this charge density has the form:

\[
E_d = \int d^2r d^2r' h(\vec{r}, t) K(\vec{r} - \vec{r}') h(\vec{r}', t),
\]

(3.36)

\[
K(\vec{r} - \vec{r}') = \frac{\mu_0 M_s^2}{2\pi |\vec{r} - \vec{r}'|^3} \left( 1 + \frac{3(x - x')^2}{|\vec{r} - \vec{r}'|^2} \right).
\]

(3.37)

\( K \) is a non-local kernel, this is responsible for the long-range nature of the dipolar interaction. Different variants for the treatment of the dipolar interaction have been widely studied \([4, 23, 32, 35, 36, 37]\).

- Surface tension’s (\( \nu_0 \)) energetic contribution has a typical form for elastic interfaces:

\[
E_{dw} = \nu_0 S_{dw} + \frac{\nu_0}{2} \int d^2r |\nabla h(\vec{r}, t)|^2,
\]

(3.38)

with \( S_{dw} \) the area of the domain wall.

- Disorder is very difficult to treat theoretically. Generally it can be modeled by the derivative of a random potential \( V(\vec{r}, h) \) which gives the local random pinning field \( \eta(\vec{r}, h) \). The difficult analytical treatment becomes, however, easier, because it has been shown \([38]\) that the particular form of \( V(\vec{r}, h) \) is irrelevant. This domain wall model takes the pinning field to be short-range correlated (or even uncorrelated) and with Gaussian distribution.

**Analytical treatment and results**

1. An infinite-ranged kernel is considered \([23, 38]\), first in the case when no demagnetizing is acting and \( H = \text{const} \) (adiabatic limit). This case corresponds to the conventional depinning transition. Around the critical depinning field the average domain wall velocity scales as

\[
v \sim (H - H_c)^\beta.
\]

(3.39)

The critical behavior of the depinning transition was studied using renormalization group methods \([38]\), and it was shown that at large length scales the critical exponents take mean-field values \([23, 35]\). Close to depinning transition the response to small variations of the applied field occurs by avalanches, whose sizes \( S \) are dis-
tributed as
\[ P(S) \sim S^{-\tau} f(S/S_0), \quad (3.40) \]
where the cutoff scales as \[ S_0 \sim (H - H_c)^{-1/\sigma} \] and is related to the correlation length \( \xi \) by \[ S_0 \sim \xi^{d+\zeta}, \] with \( \zeta \) roughness exponent. The correlation length \( \xi \) diverges as
\[ \xi \sim (H - H_c)^{-\nu}. \quad (3.41) \]
The average avalanche size also diverges at the transition point:
\[ \langle S \rangle \sim (H - H_c)^{-\gamma}, \quad (3.42) \]
where \( \gamma \) is related to \( \sigma \) and \( \tau \) by
\[ \gamma = \frac{2 - \tau}{\sigma}. \quad (3.43) \]
The distribution of avalanche durations also follows power-law at the transition:
\[ P(T) \sim T^{-\alpha} g(T/T_0). \quad (3.44) \]
The cutoff diverges as \( T_0 \sim (H - H_c)^{-1/\nu z} \), where
\[ \alpha = 1 + \frac{\nu d - 1}{z \nu}. \quad (3.45) \]

Mean-field results in 3D (this corresponds to \( \zeta = 0, z = 1, \nu = 1 \)) are \( \tau = 3/2 \) and \( \alpha = 2 \). These results apply only when \( H \) is around \( H_c \). In the case when the field is swept slowly through a hysteresis cycle, the results are \( \tau_{int} = 2 \) and \( \alpha_{int} = 3 \).

2. In the second step a model using finite driving rate and demagnetizing field was used.

The effect of the demagnetizing field is expressed in the followings through parameter \( k \equiv 4\mu_0 N M_s^2 \) (see the prefactor in the relation 3.34) where \( N \) is the intensity of the demagnetizing field. An infinite-range model was introduced, and it was shown that it should produce the same behavior as the long-range model, but it has to be much simpler to treat it theoretically. Urbach et al. [4] has already shown that the effect of the demagnetizing field is to keep the interface (domain wall) near the critical point. In Ref. [23] Zapperi et al. shows that the intensity of the demagnetization is a relevant parameter that controls the characteristic length of the avalanches, and
Figure 3.11: Distribution of avalanche sizes (a) and durations (b) in the infinite-range model after [23]

that criticality is reached only for vanishingly small demagnetization intensity.

On the other hand the driving rate has significant effect as well, it continuously changes the critical exponents. They show that the infinite-range model reproduces the ABBM results [24]. In Ref. [23] it is demonstrated that the experimentally observed [25] Brownian pinning field is not due to a long-range correlated disorder, but it is the result of the collective motion of the interface, and it is an effective description of the disorder. The average avalanche size scales as $1/k$, and the cutoff scales as $k^{-2}$. Similarly, the cutoff of the avalanche durations scales with $1/k$. These results, however, do not agree with the experiments.

Results from simulation

Besides the theoretical calculations, simulation methods and results are also given in paper [23]. They have simulated two variants of the theoretical model, both in the cases with and without a finite driving rate and demagnetizing effect.

1. The infinite range model was implemented in order to confirm its equivalence with the ABBM model. They integrated the equation of motion with Runge-Kutta method, and observed that in the direction of increasing driving rate the dynamics changes from avalanche-dominated motion toward a smoother motion for large driving rates. Next, the model was implemented on an automaton version in order to be able to simulate larger systems, and computed the avalanche size and duration distributions in the adiabatic limit and for different values of $k$ (demagnetizing parameter). Parameter $c$ characterizes the driving rate of the external field in a way that the adiabatic limit corresponds to $c \to 0$. 
Data agree perfectly with the theoretical scaling predictions. The model was simulated as a function of the driving rate as well, and shown that the scaling exponents depend linearly on the driving rate: \( \tau = 3/2 - c/2 \) (Fig. 3.11). Finally, the simulated power spectrum for different \( c \) values has a \( 1/f^2 \) decay at high frequencies and a constant part at low frequencies.

2. **Long-range models** are very hard to study. Zapperi et al. [23] implemented it on an automaton version, and considered again the cases (i) \( k = 0 \) and (ii) \( c \to 0 \) with \( k > 0 \), respectively. In the \( k = 0 \) case they increase the external field up the \( H_c \) value and compute the integrated avalanche size and duration distribution. They yield \( S^{-2} \) and \( T^{-3} \), and confirm the predictions of the theory. In the adiabatic limit the avalanche size and duration distributions yield the scaling as a function of \( k \) obtained in mean-field theory: \( \tau = 3/2 \) and \( \alpha = 2 \). The scaling of the cutoffs instead do not agree with ABBM exponents, but they scale as \( S_0 \sim k^{-1} \) and \( T_0 \sim k^{-1/2} \). Thus in order to have good data collapse the following scaling functions are valid:

\[
P(S, k) \sim S^{-3/2} g_1(Sk),
\]

\[
P(T, k) \sim T^{-2} g_2(Tk^{1/2}).
\]

We have to remark that this model is not able to account for the BN power spectrum. The model gives \( 1/f^2 \) type decay for high frequencies, but this has been observed experimentally only in the case of a single moving domain wall. The presence of many domain walls changes the power spectrum.

As a conclusion, in the presented model crucial role is played by the long-range interaction kernel which models the effect of the demagnetizing field. The theory gives results that compare well with the experiments regarding the Barkhausen jump (avalanche) size and duration distribution. The model is able to reproduce the linear dependence of the scaling exponents on the external driving field rate, as well as the scaling of the cutoff (for Barkhausen signal duration and jump-size distributions) with the demagnetizing field.

### 3.4 Stochastic Resonance in Magnetization Models

In the previous section we have presented several models aimed to explain Barkhausen noise and other phenomena related to magnetization processes of ferromagnetic materials. A generally common feature of these models is that they all study these phenomena in the
case of slow or even quasistatic driving rate of the external magnetic field. None of them
considers the dynamical response of the system under the condition of periodic driving.

It has been shown (see for example Ref. [20]) that even simple, but nonlinear noisy
systems can exhibit counterintuitive and nontrivial behavior under the effect of a weak
driving which is changing periodically in time. Noise can play a constructive role in these
kinds of systems if tuned to a nonvanishing optimal value: it can increase the degree
of coherence, can lead to the amplification of weak signals. The phenomenon is called
stochastic resonance (SR).

A special (and relatively easy to study) case of SR is found in coupled two-state
systems, for example the Ising model for ferromagnets. In an Ising system the three
needed requirements for detecting SR (see the introductory Chapter 1.) are satisfied: (i)
at zero thermodynamic temperature the free energy versus the magnetization has double-
well shape; (ii) the effect of positive temperature is a stochastic force; (iii) periodic $B(t)$
external magnetic field modulates the two minima in antiphase [39]. The magnetization
can be considered as the magnetic response function to the external driving.

Zoltán Néda and Kwan-tai Leung published a number of papers [39, 40, 41, 42] where
they extensively studied the phenomenon of stochastic resonance in Ising systems.

The Hamiltonian of the model contains the basic properties and interactions:

$$H = -J \sum_{nn} S_i S_j - h(t) \sum_i S_i,$$  \hspace{1cm} (3.48)

where $h(t) = A \sin(\omega t)$ is the periodic driving and $\sum_{nn}$ runs over the nearest neighbors
of a spin in a d-dimensional cubic lattice.

In Ref. [39, 40] Z. Néda investigated the 2D and 3D Ising model with Monte Carlo sim-
ulations. The system obeyed Glauber dynamics and it was studied in oscillating magnetic
field. The correlation function $\sigma$ between the driving field $B(t)$ and the magnetization
$M(t)$ was studied: $\sigma = |<B(t)M(t)>|$. If there exists SR in the model system, the
correlation function versus the temperature should show a maximum.

The results indicated that there is a well-defined resonance temperature at which
the correlation exhibits a maximum. The value of the resonance temperature slightly
decreases with increasing driving field amplitude, and the maximum of the correlation
strongly decreases with decreasing amplitude. As the system size is increased, the res-
onance temperature tends to a constant limiting value. In the limit of very low driving
frequency the resonance temperature approaches the critical temperature $T_c$ of the Ising
Figure 3.12: (left) Characteristic shape of the power spectrum for driving frequency \( f_s \approx 0.097 \); (right) general shape of the \( R(T)/VA^2 \) curve in one dimension for different combinations of the parameters, the continuous line is the exact result in 1D model known for the ferromagnetic-paramagnetic transition.

In Ref. [41] the authors report the results of analytical calculations. The response in the magnetization was calculated by two methods: mean-field theory with linear response approximation (in 2D and 3D case), and the time-dependent Ginzburg-Landau (TDGL) equation (in 2D). The analytic results obtained for the resonance temperature and its dependence on the frequency and amplitude of the driving field compare favorably with the simulation data. Analytical results obtained with both methods converge to simulation results in the case when \( T >> T_c \), although TDGL approach is more accurate in general.

In Ref. [42] the authors study the SR in kinetic Ising model through calculating the signal-to-noise ratio (SNR) of the magnetic response \( R_{\text{sim}} \) in 1 to 4 dimensional cubic lattice. As expected, the SNR has a very sharp peak exactly at the driving frequency for a nontrivial resonance temperature above the equilibrium critical temperature.

In the small frequency and small amplitude limit, \( R_{\text{sim}} \) is expected to scale as:

\[
R_{\text{sim}}(T; V; A) = VA^2g(T),
\]

where \( T \) is the temperature, \( V = L^d \) is the volume of the system and \( A \) is the amplitude of the driving field. Simulation data confirm this scaling law and data collapse is reasonable in all dimensions studied. As an example Figure 3.12 shows a characteristic power spectrum (left) and the shape of the scaling law in 1D for different combinations of the relevant parameter.

We believe that the SR phenomenon detected in simple models for ferromagnetism and magnetization phenomena should also appear in the more suitable models presented and discussed within this work.
Chapter 4

Spring-block Model

In the following two original models aimed to describe magnetization phenomena will be presented. Both models aim to explain the occurrence of magnetic Barkhausen noise, reproduce its statistical properties and other phenomena closely related to magnetization processes in disordered ferromagnets, like disorder-induced phase transition and stochastic resonance.

As reviewed in Chapter 3, many recent studies proved that a convenient way of treating the microscopic interactions responsible for magnetization phenomena can be done by introducing random disorder in simple magnetic models. Following this line we have introduced two one-dimensional models which include the basic microscopic ingredients that are believed to be important to qualitatively understand the phenomena.

The first model is a Burridge-Knopoff type spring-block stick-slip model [43, 44]. It is rather a toy-model for the phenomena and it is based on a simple mechanical analogy. The starting idea comes from Professor Yves Bréchet and its further development is the work of Kovács Katalin and Professor Néda Zoltán [45, 46, 47].

4.1 Presentation of the Spring-block Model

The model is essentially a one-dimensional spring-block system [45]. It is aimed to reproduce the accepted microscopic picture of domain wall dynamics for 180 degree Bloch-walls which separate inversely oriented (+| − | + | − | + . . . ) magnetic domains (Fig.4.1).

We assume that the domain walls are pinned by defects and impurities, and cannot move unless the resultant force acting on them is bigger than the strength of the $F_p$. 
Figure 4.1: Sketch of the mechanical spring-block model

pinning force. When the resulting force is greater than the pinning force, the wall simply jumps in the resulting force direction on the next pinning center. Apart of this pinning force there are two other types of forces acting on each domain wall. To understand these forces let us consider the \( i \)-th wall (which separates the \((i - 1)\)-th and \(i\)-th domain) free to move and all other walls fixed. One of the forces acting on the domain wall, \( F_H \), results from the magnetic energy of the domains \(i\) and \((i - 1)\) in an external magnetic field. Let us consider the external magnetic field as sketched in Fig.4.1. The interaction energy between one magnetic domain and the external magnetic field:

\[
W = -c_H \cdot H \cdot M, \tag{4.1}
\]

where \(c_H\) is a constant, \(H\) is the strength of the external magnetic field and \(M\) is the magnetization of the domain (the positive direction both for \(M\) and \(H\) is taken upward). Taking into account that the \((i - 1)\) and \(i\) neighboring domains are oppositely oriented, their total energy of interaction with the external magnetic field is:

\[
W(i) = W_{i-1} + W_i = -c_H \cdot H \cdot \Delta M. \tag{4.2}
\]

The quantity \(\Delta M\) (the sum of magnetizations of the neighboring \((i - 1)\) and \(i\) domains)
is related to the two domains’ length’s difference ($\Delta x = x_i - x_{i-1}$) as:

$$\Delta M = (-1)^i \gamma \cdot \Delta x, \quad (4.3)$$

where $\gamma$ is a constant relating the size of the domain with its magnetization. From (4.2) and (4.3) it results:

$$W(i) = (-1)^{i+1} c_H \gamma \cdot H \cdot \Delta x. \quad (4.4)$$

The force $F_H$ acting on domain wall $i$ can be determined considering the $\delta L$ elementary work performed by this force, when the wall is displaced by a distance $dl$:

$$\delta L = F_H \cdot dl = -dW(i) = (-1)^i c_H \gamma \cdot H \cdot d(\Delta x) =$$

$$= (-1)^i c_H \gamma \cdot H \cdot 2dl, \quad (4.5)$$

$$F_H = \frac{\delta L}{dl} = (-1)^{i+1} c_H \gamma \cdot H = (-1)^i \beta \cdot H, \quad (4.6)$$

with $\beta = 2c_H \gamma$ another constant. (In our model for the sake of simplicity we define the units such that $\beta = 1$.) This $F_H$ force is thus an external force which acts on each domain wall, its strength is linearly proportional with the external magnetic field intensity and its direction is opposite for neighboring walls. For positive values of the external magnetic field this force encourages the increase of the domains oriented in the $+$ direction, and for negative values of the external magnetic field this force tends to increase the size of the domains oriented in the $-$ direction.

A second type of force, $F_m$, acting on both sides of the domain walls, is due to the magnetic self-energy of each domain. This force tends to minimize the length of each domain. It can be immediately shown that $F_m$ is proportional with the length of the considered domain and this is why it can be modeled with elastic springs that connect neighboring walls. The $E_i$ magnetic self-energy of a magnetic domain $i$ has the form:

$$E_i = c_m M_i^2, \quad (4.7)$$

where $c_m$ is a constant. As in the previous case:

$$dE_i = c_m \cdot d(M_i^2) = -\delta L = -F_m dl, \quad (4.8)$$
and from here the $F_m$ force:

$$F_m = - \frac{dE_i}{dl} = -2c_m(M_i) \frac{dM_i}{dl} \approx -2c_m \gamma^2 \cdot x_i \frac{dx_i}{dl} = -2c_m \gamma^2 x_i = -f_m x_i. \quad (4.9)$$

The constant $f_m$ is an important coupling parameter in this model and acts as the elastic constant of a mechanical spring.

The system of the $F_p$, $F_m$ and $F_H$ forces can be now easily mapped on a mechanical spring-block model of Burridge-Knopoff type [43].

The main constituents in this mechanical model are randomly distributed pinning centers, rigid walls sitting on pinning centers (describing Bloch-walls) separating $+$ and $-$ oriented domains and springs between the walls (describing the $F_m$ forces). The strength of the pinning centers (pinning forces), $F_p$, are randomly distributed following a normal distribution. Walls can be only on pinning centers and two walls are not allowed to occupy the same pinning center. This constraint implies that the number of magnetic domains and domain walls are kept constant and are thus a-priori fixed. Domains cannot totally disappear and new domains cannot appear during magnetization phenomena. The elastic springs are ideal with zero equilibrium length and with the tension linearly proportional with their length. The tension in the elastic springs will reproduce the $F_m$ forces. Beside the pinning forces and the tensions in the springs there is an extra force acting on each wall. The strength of this force is proportional with the exterior magnetic field’s intensity, it has the same value for all walls but its direction is inverse for $+|-$ and $-|+$ walls. This force will reproduce the $F_H$ forces.

It has to be noticed that in the absence of applied field and pinning centers the walls at equilibrium will be equally spaced. Since we impose the number of walls to be constant, this means that the average wall spacing will be given by the classical expression corresponding to the minimization of the magnetostatic energy and of the total wall energy.

The model as it stands is grossly simplified: in a real system the walls are 2D, they have an anisotropic stiffness, so they may be described by the dynamics of their trace (1D) on the plane perpendicular to the easy magnetization axis. The depinning of this line is unlikely to occur as a whole, but it will probably propagate all along the line. Because of all these simplifications the present model cannot be expected to be directly connected to microstructural features (such as the density and strengths of the physical pinning centers), neither to be quantitative. The present model is important mainly for
pedagogical purposes, to define the minimum ingredients necessary to capture the various
power laws observed. In addition, the relative importance of the pinning potential (via
$F_p$, the strength and $N_p$, the number of pinning centers) and of the coupling between walls
(via $F_m$) can be qualitatively investigated.

4.1.1 Dynamics

The dynamics of this model is aimed to reproduce real magnetization phenomena. First $N_p$ pinning centers are randomly distributed on a fixed length ($L$) interval, and their strengths are assigned. Than a fixed $N_w$ number of walls are randomly spread over the pinning centers ($N_w \ll N_p$) and connected by ideal springs. Neighboring domains are assigned opposite magnetic orientation. The external $F_H$ force is first chosen zero (corresponding to $H = 0$), and we let the system relax to an equilibrium configuration. To achieve this we calculate the resultant of the $F_m$ forces on each wall. If the strength of the pinning force acting on one wall is smaller than the force resulting from the tension of the springs attached to it, the wall will jump in the direction of the resultant force on the next pinning site. However, if this pinning site is already occupied, the wall remains in its original position. We assume that the time needed for the system to achieve equilibrium is zero. It is important to note that one event (jump) can trigger many other events leading to avalanche-like processes. The above dynamics is continued until the equilibrium is satisfied for each domain wall. The order in which the position of the walls is updated is random.

Once the initial equilibrium configuration is reached we begin to simulate the magnetization phenomenon. The value of the $F_H$ external force is increased step-by-step (corresponding to an increasing $H$ magnetic field intensity), and for each new $F_H$ value an equilibrium position of the system is searched. In equilibrium the magnitude of the resulting force on each wall should be less than the pinning force acting on that wall. In each equilibrium configuration we calculate the total magnetization of the system as:

$$M = \sum_i l_i \cdot s_i. \quad (4.10)$$

where $l_i$ is the length of domain $i$, and $s_i$ is its orientation: $+1$ for positive orientation, and $-1$ for negative orientation. We increase $F_H$ until no more walls can move and the magnetization reaches its maximal value. Starting from this we decrease step-by-step the value of $F_H$, and for each new value the equilibrium configuration is again reached.
and the total magnetization computed. The system is driven until oppositely oriented saturation. From here we increase again the value of $F_H$ and many hysteresis cycles are simulated. Throughout the whole simulation we assume that the process of magnetization is performed at a rate slow enough so that at each step the system can reach a position of equilibrium in a sort of quasistatic magnetization process.

During the simulation we are monitoring the variation of the magnetization focusing on the shape of the hysteresis loop, jump-size distribution, power spectrum, Barkhausen signal duration and signal area distribution functions.

The Barkhausen signal is given by $\frac{dM}{dt}$ and it is proportional with an electric voltage that would be induced in a detecting electromagnetic coil. Under the assumption of a quasistatic process performed at a constant rate of increase of the applied magnetic field $H$, the derivatives with respect to time which enter the generation of Barkhausen noise are proportional to derivations with respect to $H$: $\frac{dM}{dH}$. This means that the magnitude of the Barkhausen jumps is not to be understood as absolute values, but would depend on the rate of increase of $H$.

We determined the power spectrum ($P(f)$) of the obtained Barkhausen signal by using a Fast Fourier Transform (FFT) of the Barkhausen jumps. Since in our simulations there is no real time, the frequency is also not realistic, and it is defined solely by $dH$ which can be considered in a first approximation to be constant in time. Thus, we emphasize that the power spectrum determined from this simulation should be viewed only under these constraints.

We also study the shape of the histograms for Barkhausen signal duration distribution $(g(d))$ and signal area distribution $g(A)$. In terms of our simulation the signal duration measures the number of consecutive $dH$ steps when Barkhausen jumps occur ($\Delta M/dH$ is nonzero). Signal area is also related to this quantity: it represents the area under $\Delta M/dH$ versus $H$ for a nonzero $\Delta M/dH$ sequence.

The parameters of the model are: $N_p$ – the number of pinning centers; $N_w$ – the number of Bloch-walls ($N_w \ll N_p$, the ratio $N_p/N_w$ is important); $L$ – the geometrical size of the system, where $N_p/L$ defines the density of the pinning centers in the sample; $f_m$ – the coupling constant between the neighboring domain walls (corresponds to the elastic constant in the case of coupled springs); the $dH$ – driving rate of the external magnetic field (change in $H$ for one simulation step); $\sigma$ – the standard deviation of the normally distributed $F_p$ pinning forces, called disorder.

We use rigid boundary conditions: the first Bloch-wall compulsory occupies the first
pinning center, the last wall occupies the last pinning center, and these bounding walls cannot move. This constraint means that the geometrical size of our model system doesn’t change during the simulation. As we already mentioned the number of Bloch-walls and magnetic domains are also fixed within this model, domains can shrink or grow, but they cannot appear or disappear during the simulation. This constraint is also a serious drawback of the model, since in real magnetization phenomena domains can disappear or appear, and thus the number of interfaces is not constant.

## 4.1.2 Influence of the free parameters

Before presenting the simulation results let us discuss here in what range the free parameters have to be chosen in order to get reasonable results. We will use adimensional units for the forces, their strengths are determined relatively to the adimensional strength of the pinning forces. The $f_m$, $\beta$, $H$ and $x$ quantities are defined through equations (4.6) and (4.9) and they are also considered to be adimensional.

- The value of $N_p/N_w$ should be high enough in order to have jumps on many possible length-scales. However, as $N_p$ increases the simulation is more and more time consuming, since many small jumps and thus many intermediate equilibrium positions are available. On the other hand, if we choose $N_w$ small the model becomes unrealistic, due to the small number of simulated magnetic domains. Pondering all these considerations we performed our calculations with $N_p/N_w$ between 5 and 100, varying $N_p$ between 1000 and 10000.

- The parameter $f_m$ acts in our spring-block model like the elastic constant of the springs, its value determines the value of attractive forces between neighboring walls. Its effect on the BN noise is somehow similar with the effect of changing $N_p/N_w$ when $L$ is fixed. For high values of $f_m$ (equivalent with low $N_p/N_w$) one will have big avalanches and the equilibrium configuration will be hardly reached. The hysteresis loops will have unrealistic shapes. For very low values of $f_m$ the motion of Bloch-walls will be very smooth and the jumps will be limited on short length-scales. In our simulation we fixed this parameter to an intermediate value, chosen to be $f_m = 10$ in the considered adimensional units.

- The driving rate $dH$ is also important for all the results given by the model. The best would be to choose $dH$ as small as possible in order to reproduce the continuous dynamics. Choosing $dH$ very small makes the simulation very slow, choosing
however a too large step will introduce unrealistic jumps. Large values of \( dH \) would be appropriate also for describing the experimental results obtained for high driving frequencies. In the experiments it is reported however [3, 11, 12, 36] that they used very low frequencies (< 1 Hz) for the driving field. Taking into account all these arguments we made our simulations with as small \( dH \) steps as allowed by the available computational resources (\( dH = 0.001 \)).

- **Influence of the disorder \( \sigma \) and the pinning center density \( N_p/L \)

As it is known from previous studies on random field and random bond Ising models [14, 15, 18, 30] the amount of disorder in the considered model has a crucial importance on the statistical properties of the obtained magnetization noise. For low disorder and very strong disorder values the jump-size distribution does not exhibit a scaling property. Usually there is a critical amount of disorder for which the jump-size distribution has a power-law decay. In our model the amount of disorder can be controlled in two ways, either by varying the density of pinning centers (\( N_p/L \)) or by changing the standard deviation \( \sigma \) of their strength’s distribution function. We investigate both of these methods and expect qualitatively similar results. Because of the crucial importance of the disorder as a tuning parameter in the model, we dedicate a whole section (section 4.3) for the study of its effect.

The relevant distribution functions were obtained by averaging on 100 independent configurations (different initial states for the walls and pinning centers). For illustrating that the obtained distributions and scaling exponents are not strongly altered by finite-size effects (i.e. the behavior characteristic for the thermodynamic limit is reached) we present results for three different \( L \) system sizes.

All the distribution functions we have computed for the statistics of the Barkhausen noise are normalized and defined as

\[
g(y) = \frac{N(y, y + dy)}{N_t \cdot dy},
\]

where \( N_t \) denotes the total number of occurrences, \( N(y, y + dy) \) is the number of occurrences with sizes between \( y \) and \( y + dy \), \( y \) is the quantity in focus which can be jump-size (\( s \)), signal duration (\( d \)) or signal area (\( A \)).
4.2 Simulation Results

During the simulations we focus on the variation of the magnetization, constructing the shape of the hysteresis loop and jump-size distribution. The hysteresis loop is the history-dependent relation between the magnetization $M$ and the external magnetic field $H$ when the value of $H$ is successively increased and decreased. The jump-size distribution ($g(s)$) is the distribution function of the abrupt jumps in $M$ throughout many hysteresis loops.

For different disorder levels in the system characteristic hysteresis loops and corresponding jump-size distributions are plotted on Fig. 4.2.

In a wide range of parameters the shape of the obtained hysteresis curves satisfies our expectations and fulfills all the requirements for real magnetization phenomena. On these curves one can detect many discrete jumps with different sizes, thus the model exhibits BN. In addition, when the sample is driven consecutively through many hysteresis cycles the magnetization curves do not follow exactly the same path, although the parameters of the simulation were unchanged (as shown on Fig. 4.2(c)). The qualitative shape of the hysteresis curve is quite stable for a wide range of the free parameters. For low disorder in the system (figure (a), $\sigma = 0.1$) the hysteresis loop has rectangular shape with only small number of jumps at the beginning of the demagnetization. These small jumps are due to the weak influence of the pinning centers, but after a short unstable region the system reaches equilibrium. As the external magnetic field is strong enough, the sample’s magnetization reverses abruptly from positive to negative saturation through a huge avalanche which spans the whole system. This kind of hysteresis curve is characteristic to a class of ferromagnetic materials which contain small amount of impurities and were annealed in such a way to inhibit the formation of internal stresses. As the amount of disorder is increased in the model system, (figures (b), (c), (d) with $\sigma \geq 0.3$) the hysteresis curve changes qualitatively. Large number of magnetization jumps of very different sizes occur during a cycle, and the hysteresis loops enlarge significantly with increasing disorder. This feature indicates that our model reproduces suitably the fact that pinning centers act as potential barriers which block the free movement of the domain walls. Thus, as the amount of quenched disorder increases in the model, stronger external driving field is needed to induce magnetization reversal. Magnetic materials treated with methods favoring the formation of internal stresses show similar hysteretic behavior.

The jump-size distribution histogram in our simulations corresponds to the avalanche
Figure 4.2: Hysteresis loops (left column) and corresponding jump-size distributions (right column) for different amount of disorder in the system. Figures (a) are for $\sigma = 0.1$, figures (b) are for $\sigma = 0.3$, figures (c) are for $\sigma = 1$ (three consecutive loops) and figures (d) stand for $\sigma = 10$. The power-law fit for the jump-size distribution on figure (b) indicates a scaling exponent $-0.97$. Other parameters of the simulations are: $N_p/N_w = 100$, $N_p = 7000$, $L = 7$, $f_m = 10$ and $dH = 0.001$. The inset on the hysteresis (a) enlarges the region in the red box.
size distributions from experiments and it is the most relevant distribution for the characterization of the Barkhausen noise.

The right columns on Fig. 4.2 show the jump-size distributions corresponding to the hystereses in the left column. All histograms are on log-log scale. Fig. 4.2(a) shows the case when the amount of quenched disorder is very low in the system. The hysteresis consists of a few small jumps at the beginning and two spanning avalanches which switch the sample from positive to negative and again positive saturation. Thus, on the jump-size distribution histogram we cannot detect any scaling tendency. This regime is called "supercritical". Figure (b) corresponds to $\sigma = 0.3$ disorder. The jump-size distribution histogram is a clear power-law over four decades with exponent 0.97. This exponent does not modify significantly if we increase the system size, which means that there is no detectable finite size effect. The fact that our simulation results indicate a power-law distribution in the thermodynamic limit means that jump- (avalanche-) sizes exhibit scale-invariance, a feature that is expected to be common for all types of crackling noises. Power-law distribution may be an indicator of criticality in a system. In the vicinity of continuous phase transitions relevant physical quantities have also power-law behavior. In the next section (4.3) we will examine in detail the possible occurrence of a disorder-driven phase transition in this model. The parameter regime where we have power-law jump-size distribution is called "critical". The shape of the hysteresis curve changes qualitatively compared to case (a): the magnetization reversal is not abrupt anymore, but we encounter large number of jumps with sizes ranging over four decades of magnitude. Increasing further the disorder (Figures (c) and (d)) the jump-size distributions do not follow power-law, small avalanches dominate and the prevalence of large jumps decreases faster than power-law. The system behaves "subcritically".

In the critical regime the power spectrum indicates a perfect $1/f$ type noise independently of the system size (Fig. 4.3.). We have also checked that in the subcritical regime ($\sigma > 1$), the power spectrum of the BN signal shows almost a perfect white noise over several decades.

We mention here again that this power spectrum has to be treated with care since we do not have real time in simulations, and thus we cannot define a proper frequency. Time evolution is substituted with the driving rate $dH$ of the external field and it is considered that equilibrium is reached for each simulation step. We have plotted thus the "power spectrum" in terms of the $1/dH$ – type "frequency", assuming the unit time as the time
Figure 4.3: Power spectrum of the simulated BN in the critical regime, results for three different system sizes. The continuous line indicates a power-law with exponent $-1$. Other parameters of the simulations are: $N_p/N_w = 100$, $N_p/L = 300$, $f_m = 10$ and $dH = 0.001$.

needed to change the external magnetic field by $dH$.

On Fig. 4.4. we plotted for different system sizes ($L$) the signal area distribution function in the critical regime. The same parameters were used as in the previous graphs ($N_p/N_w = 100$, $f_m = 10$ and $dH = 0.001$). The curves suggest a power-law behavior over three decades of data. The exponent of the fitting power-function is around $-1$. No important finite size effects can be detected. In contrast with these results, in the subcritical regime the signal area distribution function does not show a clear scaling property, and on a log-log scale has a constantly decreasing negative slope.

4.3 Disorder-driven Phase Transition in the Spring-block Model

As it was shown in the previous section 4.2 the model exhibits a fair statistics for Barkhausen noise. It has also the necessary requirements for the occurrence of a fluctuationless phase transition. The characteristic time-scales of the model system are: $\tau_a = 0$ (instant response), slow driving rate and zero temperature, which means that $\tau_{th} \rightarrow \infty$. Thus the relation $\tau_a << \tau_{dr} << \tau_{th}$ holds. Here we investigate the possibility of a disorder-driven phase transition which should appear while the amount of disorder is varied in the system.

As it is known from previous studies on RAIM, RFIM and RBIM [13, 14, 15] the
amount of disorder in the model has a crucial role on the statistical properties of the obtained jumps in magnetization. For low and very strong disorder values the jump-size distribution does not exhibit scaling property. Usually there is a critical amount of disorder for which the jump-size distribution has power-law decay.

In order to investigate properly the existence and characteristics of the disorder-induced phase transition in the spring-block model, we need to define accurately the order parameter and the tuning parameter. According to Refs. [13, 15] we define as order parameter the relative size of the largest avalanche $S_{\text{max}}/L$ and the tuning parameter is the disorder. This phase-transition should be observable by studying the size of the largest avalanche as a function of the amount of quenched disorder introduced by the randomly distributed pinning centers. The order parameter is computed by averaging over several hundreds of different realizations of the quenched disorder, keeping all parameters of the model fixed.

In the present model the disorder is induced by the randomly distributed pinning centers. The amount of disorder can be controlled in two manners: either by varying the average distance, $L/N_p$, between the pinning centers (inverse of the density of the pinning centers $N_p/L$) or by changing the standard deviation $\sigma$ of their strength’s distribution. We use both of these methods and expect qualitatively similar results. The parameters are the same as given in the caption of Fig. 4.2 and we vary $\sigma$ or $N_p/L$.

First, by keeping the density of the pinning centers constant, the order parameter,
Figure 4.5: Relative size of the largest avalanche as a function of the disorder for fixed pinning center density \(N_p/L = 1000\) and different system sizes: \(L = 1, L = 2, L = 3, L = 4, L = 6\) and \(L = 7\). Other parameters of the simulations are: \(N_p/N_w = 100, f_m = 10\) and \(dH = 0.001\).

\[S_{\text{max}}/L, \text{versus } \sigma\] is studied. For increasing system sizes the simulation results are plotted on Fig. 4.5. The shape of the curves indicates that the expected disorder-induced phase transition occurs. We can read from the results that for low values of the disorder \((\sigma < \sigma_{\text{crit}})\) huge avalanches arise which sweep through the whole system, the magnetization reverses abruptly at a certain intensity of the driving field. This phenomenon was already clearly observable when we have plotted the hysteresis curve in the supercritical regime (see Fig. 4.2(a)) and is called the ordered phase.

There is a critical amount of disorder \(\sigma_{\text{crit}}\) where the \(S_{\text{max}}/L\) versus \(\sigma\) curves have their inflection point. In the neighborhood of \(\sigma_{\text{crit}}\) the avalanche sizes will drop drastically and the distribution of avalanche sizes obeys power-law. Exactly this was shown to happen on Fig. 4.2(b): (i) the relative size of the greatest avalanche decreases; (ii) large number of jumps are visible; (iii) the sizes of these jumps range over several orders of magnitude.

Fig. 4.5 suggests that by increasing the system size the curves \((S_{\text{max}}/L \text{versus } \sigma)\) show similar behavior, the location of the inflection point does not move and the two phases become more and more distinct.

For high amount of disorder \((\sigma > \sigma_{\text{crit}})\) there is no possibility for spanning avalanches to occur, this is called the disordered phase. The relative size of the largest avalanche in this case is much smaller. For an infinite system one would expect that the relative size of the non-spanning avalanches should approach zero. Due to the fact that computer simulations were performed with finite system sizes, this quantity is naturally greater
than zero. It is quite simple to prove that for infinite system sizes and for high values of \( \sigma \) there is a clear tendency for the \( S_{\text{max}}/L \) quantity to approach zero. In order to do this we considered the case of \( \sigma = 0.8 > \sigma_{\text{crit}} \) and plotted the value of \( S_{\text{max}}/L \) as a function of system size. Results are shown on Fig. 4.6 and are well fitted by a power-law with exponent \(-0.76\). The power-law nature of this curve supports our expectation that for \( \sigma > \sigma_{\text{crit}} \) indeed the order parameter \( S_{\text{max}}/L \) goes to zero as the system size tends to infinity.

Using polynomial fits we located the inflexion point of the \( S_{\text{max}}/L \) versus \( \sigma \) curves. This is indicated on the Fig. 4.5 by dashed lines. Results suggest that regardless of the system size the critical amount of disorder for this particular pinning center density is the same, namely \( \sigma_{\text{crit}} = 0.28 \pm 0.02 \). The corresponding relative size of the percolating avalanche is: \( S_{\text{max}}/L = 0.59 \pm 0.03 \). This avalanche size at criticality is in good agreement with the result reported by Vives and Planes [13]. By the other hand these numerical results confirm that indeed the hysteresis loop and corresponding jump-size distribution histogram on Fig. 4.2(b) is very near to the critical point.

We have also studied the disorder-induced phase transition as a function of the average distance \( (L/N_p) \) between the pinning centers. In the simulation the number of pinning centers were kept constant \( (N_p = 1000) \) and the size of the system, \( L \), varied. The phase transition should be observable by plotting the relative size of the maximal avalanche

Figure 4.6: Relative size the largest avalanche at \( \sigma > \sigma_{\text{crit}} \) disorders for fixed pinning center density \( (N_p/L = 1000) \) and different system sizes: \( L = 0.5, L = 1, L = 2, L = 3, L = 4, L = 5, L = 6 \) and \( L = 7 \). Other parameters of the simulations are: \( N_p/N_w = 100, f_m = 10 \) and \( dH = 0.001 \). The dashed-line indicates the best power-law fit.
(order parameter), $S_{\text{max}}/L$, versus $L$ (which is proportional with $1/$density). Results for three different $\sigma$ values are given on Fig. 4.7.

The graphs from Fig. 4.7 indicate that disorder-driven phase transition is present again and clearly observable for a wide range of $\sigma$ values. As the strength of the disorder in the pinning force values is increased ($\sigma$ is increased), the phase transition occurs for lower and lower pinning center densities, i.e. $L_{\text{crit}}$ increases with increasing $\sigma$ values. This result is not surprising at all, since the total amount of quenched disorder in the system depends linearly both on $\sigma$ and $N_p/L$. An immediate estimate for the total amount of disorder would be: $\sigma N_p/L$, which would yield the:

$$L_{\text{crit}} \propto \sigma \quad (4.12)$$

scaling law for the $L_{\text{crit}}$ phase transition point. The transition point $L_{\text{crit}}$ can be estimated from the coordinates of the inflection point in the $S_{\text{max}}/L$ versus $L$ curves. After studying the disorder-driven phase transition for several $\sigma$ values the validity of (4.12) can be verified. On Fig. 4.8 we plotted the computed $L_{\text{crit}}$ values as a function of $\sigma$. There is a clearly visible linear trend with slope $= 2.8$ and this proves our scaling hypothesis.
Figure 4.8: Scaling of the inflection point location (obtained from the $S_{\text{max}}/L$ versus $L$ curves) for different $\sigma$ values. Data indicate a linear scaling with slope 2.8. Other parameters of the simulations are: $N_p/N_w = 100$, $N_p = 1000$, $f_m = 10$ and $dH = 0.001$.

Figure 4.9: Relative size of the largest avalanche as a function of the disorder in the one-dimensional RFIM for different system sizes: 1000, 2000, 3000, 5000 and 10000 spins. Coupling constant $J = 1$, driving rate $dB = 0.001$. 
4.4 Overview of the Spring-block Model

The simple one-dimensional spring-block model presented in this chapter is successful in qualitatively reproducing the statistics of the magnetic Barkhausen noise. The model captures the main elements of the microscopic dynamics for the phenomenon and in spite of its gross simplification, it contains all the necessary ingredients to describe the statistics of the BN. The model is also suitable for pedagogical purposes and can be easily implemented on computer.

The model is able to reproduce several basic properties of magnetization phenomena:

The shape of the simulated hysteresis loops is in good agreement with those obtained experimentally. One can observe many Barkhausen jumps with various sizes, just as it is expected from the experimental results. We can also learn from the graph 4.2 that the hysteresis loops do not follow the same path when the system goes through several cycles. This fact proves that this model can reproduce the irreversible character of the magnetization–demagnetization process.

Experiments measure a wide variety of magnetic materials. Some of them have crystalline or polycrystalline structure, others are amorphous; some materials were annealed under different conditions which modify significantly the type, amount and spatial distribution of pinning centers. Thus, the comparison between the experimental and our simulation results can be only qualitative.

In agreement with the previously used models [13, 14, 15], as a function of the amount of disorder quenched in the system our model also suggests the existence of three different regimes: subcritical, critical and supercritical. In the vicinity of a critical disorder value the distribution of the avalanche sizes follows power-law over several decades of magnitude.

As already emphasized, the power spectrum obtained within our model should be treated with care. The reason for this is that we do not have real time in our simulations and the equilibration of the system is instantaneously in real time.

Despite of these concerns, our result which suggests a 1/f type noise in the critical regime (exponent −1) and a white noise (exponent ≈ 0) in the subcritical regime is in agreement with several results. The obtained white noise in the subcritical regime is in agreement with O. Narayan’s [32] prediction for one-dimensional systems and the obtained 1/f type noise in the critical regime supports the experimental results of Spasojevic et
al. [3] and Plewka et al. [48].

The simulation results for the signal area distribution indicate a scaling behavior with an exponent around the value $-1$. Several experimental results proved the power-law nature for this distribution function [3, 4, 11, 36].

For the signal area distribution experiments obtained power-laws with exponents ranging from $-1.7$ to $-1.8$ [3], $-1.33$ [4], $-1.74$ to $-1.88$ [11] and $-1.23$ to $-1.35$ [36]. The measured value seems to be strongly dependent on the material measured and experimental setup. Our simulations predict a clear scaling on three orders of magnitude with an exponent $-1$, somewhat smaller than all the obtained experimental results.

A very important result of the spring-block model is that it is able to show a disorder-driven second order phase transition. The quenched disorder was introduced by randomly distributed pinning centers, acting as static friction forces on the domain-walls. The amount of disorder could be controlled either by changing the density of the pinning centers $N_p/L$, or by varying the standard deviation $\sigma$ of their strength. The system was driven along many hysteresis loops and the statistics of the jumps in magnetization (avalanche sizes) was computed. As relevant order parameter the relative size of the maximum avalanche (maximum jump in magnetization) $S_{max}/L$, was considered. Disorder-induced phase transition was observed both as a function of $\sigma$ and $N_p/L$. At criticality (critical amount of disorder) we found that the statistics of the jumps (avalanches) in magnetization follows scaling properties. For low amount of disorder (smaller than the critical amount) the system evolves with big, so-called spanning avalanches $S_{max}/L \rightarrow 1$. For high amount of disorder (bigger than the critical amount) the system evolves with many small avalanches $S_{max}/L \rightarrow 0$. It is also shown that criticality can be reached for several parameter values, characterized by a scaling law between $\sigma$ and the density of the pinning centers.

The disorder-induced phase transition found in our simple one-dimensional mechanical model is very similar with the one found in earlier higher dimensional models aimed to describe magnetization phenomena and Barkhausen noise (RAIM [13], RFIM [14], RBIM [15]). Here we have shown that the fluctuationless phase-transition can be observed also in one-dimensional out-of-equilibrium systems.

We emphasize a few advantages of using this spring-block model relative to the other
well known models, like the RFIM [14, 30]. As one can see from the Hamiltonian (3.10), the RFIM considers only three basic interactions: ferromagnetic coupling, interaction with the external driving field and the disorder introduced through the random fields. RFIM like many of the previous models does not account for the demagnetization effect.

A fundamental difference between RFIM (or other spin-type models) and the spring-block model is that the latter shows disorder-induced phase transition for a non-trivial disorder amount even in the simple one-dimensional case. Previous work on RFIM (see for example [30]) report the existence of such kind of phase transition in three and higher dimensions. In the RFIM in one dimension a transition-like curve is obtained in finite systems (Fig. 4.9). A finite-size analysis suggests however, that the critical amount of disorder tends to infinity for infinite system sizes. Moreover, the shape of the $S_{\text{max}}/L(\sigma)$ curves indicates a less and less obvious transition-point (smaller slope) while the system size is increased. This behavior is different from the one observed in our spring-block model (Fig. 4.5), where the transition point is stable and in the vicinity of the critical amount of disorder the $S_{\text{max}}/L(\sigma)$ curves get sharper as the system size is increased.

Despite of the encouraging results the model is far from being perfect. One very important feature is the absence of the temperature as parameter. Also, there is no real time in simulations, and only the value of the $dH$ magnetization step determines the rate at which time evolves in our simulations. The model does not account for the pinning mechanism and the strength of the pinning forces. It is an oversimplified one-dimensional approximation for the complex three-dimensional domain topology.

Seemingly the most serious problem of the present model is that the number of domain walls is a priori fixed and domains cannot appear or disappear during the dynamics. In a second model presented in the next chapter this deficiency is completely solved.

There are many other complex and more specialized models, elaborated for different experimental setups and different materials [26]. Our model cannot, and does not intend to compete with these ones in describing the experimental results. The stated aim of the introduced model is to give a simple and somehow general model for the magnetization phenomena, especially the Barkhausen noise, that can be successfully used for pedagogical and simulation purposes.
Chapter 5

Spin Hamiltonian Model with Random Pinning Centers

The second model [49] going to be presented in this chapter is a natural continuation and development of the spring-block model discussed in the previous chapter.

Both models are one-dimensional and very simple ones. The aim of these models is to reproduce (at least qualitatively) real magnetization phenomena and the most important scaling properties of Barkhausen noise. This new model is designed for overcoming two main deficiencies of the previous model: (i) the lack of the explicit form of the Hamiltonian which would govern the dynamics of the system, and (ii) the predefined constant number of magnetic domains.

5.1 Presentation of the Spin Hamiltonian Model

The magnetic system is modeled here by a one dimensional spin chain. Spins may have two states: up or down oriented ($s_i = \pm 1$). Ferromagnetic domain structure is taken into account in the following simple way: neighboring spins with the same orientation are members of one magnetic domain. Between two opposite oriented spins is considered to be a 180 degree Bloch-wall that separates two domains. Wherever in the model there are inversely oriented neighboring spins, they will automatically belong to different domains and between them is considered to be a Bloch-wall.

Defects and impurities that are present in the crystalline structure of real magnetic systems in this model are taken into account again through pinning centers. The pinning centers act as potential barriers which block the free movement of the Bloch-walls until
CHAPTER 5. SPIN HAMILTONIAN MODEL WITH RANDOM PINNING CENTERS

5.1.1 Hamiltonian

The model system constructed in such way defines a mesoscopic model for magnetic domains. In the Hamiltonian of the system the basic interaction terms are included: Ising-type exchange nearest-neighbor interaction, pinning, interaction with the external driving magnetic field and magnetic self-energy.

The Hamiltonian is defined thus as:

\[ H = -J \sum_{i=1}^{N_s} S_i S_{i+1} + \sum_{i=1}^{N_s} (S_i S_{i+1} - 1) T_i - B \sum_{i=1}^{N_s} S_i + 
\]

\[ + K \left( \frac{\sum_{i=1}^{N_s} \sum_{j=i}^{N_s} \prod_{k=i}^{j} (1 + S_i S_k)}{2} \right) - \frac{N_s}{2}. \]  \hspace{1cm} (5.1)

The four terms in the Hamiltonian include all the interactions that are supposed to govern the dynamics of a domain wall. The first term represents the Ising-type exchange interaction between neighboring spins with \( J > 0 \) (ferromagnetic system) coupling constant. According to this term an energetically favorable situation is when all spins have the same orientation. This term alone would drive the system toward a single-domain state. We define the units such that \( J = 1 \).

The second term reproduces the effect of the pinning centers. The \( T_i > 0 \) factors characterize the strengths of the pinning centers. Their distribution follows a Gaussian distribution and the standard deviation of the distribution (\( R \)) is called disorder. This term decreases the total energy only if neighboring spins have inverse orientation. Whenever two neighboring spins have the same orientation the pinning term vanishes. In this manner the pinning centers act only on domain walls by blocking their free movement. The pinning term is essential for reproducing the hysteretic behavior.

The third term in the Hamiltonian is a classical one and describes the interaction of spins with the external driving magnetic field (\( B \)). It is energetically favorable when spins have the same orientation as the external field has.

The last term in the Hamiltonian represents the magnetic self-energy of each domain, with a \( K > 0 \) constant. This energy is proportional with \( M_j^2 \), where \( M_j \) is the magnetization of domain \( j \). In this model we define the magnetization of a domain to be
proportional with its length, e.g. with the number of spins contained in a certain domain. This term alone would create as many (and consequently as short) domains as possible. Each domain has a positive contribution to the total energy. This contribution is proportional with the length of the domain.

Since this term is expressed in a quite complicated manner with the use of the spin variables, let us see in laymen terms what does this last term in the Hamiltonian mean and how does it work. Take the first spin and check which is the first opposite oriented spin in the array. At this point the next domain begins. Let the length of the \( j \)th domain (i.e. the number of spins in the domain) be \( l_j \). This domain contributes to the total Hamiltonian with a self-energy proportional to \( l_j^2 \). Indeed, according to the self-energy term the \( j \)th domain contributes with \((1 + 2 + ... + l_j) - l_j = \frac{l_j^2}{2}\). The product in this term actually detects the end of a domain by becoming zero whenever it meets the first time an opposite oriented spin. This way the domains can be also counted. Let the number of domains be \( n \), then the self-energy term reads as follows:

\[
E_{\text{self}} = K \sum_{j=1}^{n} \frac{l_j^2}{2}.
\]  

(5.2)

5.1.2 Dynamics

The model is aimed to reproduce real magnetization phenomena in a simple way. Its dynamics is governed by the tendency to reach the minimum energy state (called equilibrium) under certain parameter values and externally imposed conditions. In the first variant it is a zero-temperature model where the dynamics is a classical one-spin energy minimization dynamics which goes through the following steps:

First an initial random configuration of the spins is generated. The size of the system is given by the \( N_s \) number of spins. The external magnetic field has zero intensity yet. The initial value of the Hamiltonian is computed. One spin is flipped. This has to be always the spin that causes the greatest decrease in the energy. We continue flipping spins in such manner until the Hamiltonian cannot be decreased further with this single-spin method. This state is called equilibrium. We note here that this state is not necessarily the global energy minimum, rather a metastable state, a local minimum on the free energy landscape. Flipping one spin at a time cannot induce large enough changes in energy which would allow the system to overcome possible high energy barriers and reach the global minimum. With this single-spin-flip dynamics the system is then driven through metastable states toward saturation magnetization. Experimentalists also confirm this
Once the initial equilibrium is reached we start to simulate the magnetization phenomena. We increase the intensity of the external magnetic field with small $dB$ steps. The new value of the Hamiltonian is computed, equilibrium state is found with the single-spin-flip dynamics. This is taken to be one simulation step. We repeat this procedure until the sample under study is more than 90% saturated. (A system is completely magnetized (saturated) when all the spins are oriented in the direction of the external driving magnetic field.) We do not drive this model system into complete saturation because this needs very strong external magnetic fields and too many simulation steps without relevant changes in the system’s behavior. Demagnetization process begins: the external magnetic field is decreased step by step until opposite oriented saturation. The model system is driven through one or more complete hysteresis loops.

The magnetization is performed at a slow rate, so that at each step equilibrium is reached. Under the assumption of a quasistatic process, when the applied external magnetic field $B$ is changed at a low constant rate, all the derivatives with respect to time are proportional with derivatives with respect to $B$. Throughout this work all time derivatives will be replaced by derivatives with respect to $B$.

The single-spin-flip dynamics will change the number and length of magnetic domains through the following processes: (1) If the flipped spin was the first member of a domain (e.g. last member of the neighboring domain) the length of the domain increases, because the number of the spins in the domain is increased by one (consequently the size of the neighboring domain decreases); (2) one existing domain can split up in two parts by a nucleated new domain of unit length, if the flipped spin is inside an old domain. In such manner the number of magnetic domains is increased by two; (3) if the flipped spin was the member of a unit length domain, than this domain disappears and the neighboring ones will unite. In such manner the number of domains is decreased by two.

From the dynamics sketched above it results that the number of Bloch-walls and magnetic domains is not a-priori fixed anymore, but it can vary according to energetically favorable states. Thus the number of walls in the system is determined solely by the minimum energy criterion under given parameters and externally imposed conditions. In this manner the main deficiency of the spring-block model, the fixed number of domains, is solved.

During the simulation we track the variation of the magnetization and we study the
shape of the hysteresis loops, the corresponding jump- (avalanche-) size distribution, power spectrum, signal duration distribution and signal area distribution.

All the distribution functions we have computed for the statistics of the Barkhausen noise are normalized and defined according to formula (4.11). Statistics over 1000 different initial configurations and in all cases complete hysteresis loops are presented.

As seen in the case of the spring-block model, the hysteresis loop is defined as the relation between the external driving magnetic field \( B \) and the magnetization \( M \) when the system is driven through successive magnetization and demagnetization processes. Barkhausen jump-size distribution \( g(s) \) is the distribution function of the sizes of discrete magnetization jumps throughout many hysteresis cycles. Experimentally Barkhausen signal is given by \( \frac{dM}{dt} \). Taking into account the substitution presented in the above paragraph, in our model Barkhausen jumps are defined like \( \frac{dM}{dB} \). Thus the simulated jump-sizes are not absolute values because they might depend on the external magnetic field driving rate. All our simulations were made with one well defined driving rate, namely: \( dB = 10^{-3} \). This rate is low enough to correspond for quasistatic driving.

Signal duration \( g(d) \) and signal area \( g(A) \) distribution functions (histograms) were also studied. In terms of this spin model signal duration is defined as the number of consecutive \( dB \) steps (= simulation steps) when Barkhausen jumps occur (the length of a \( \Delta M/dB \neq 0 \) sequence). Signal area measures the area under the nonzero \( \Delta M/dB \) sequence versus \( B \).

Power spectra of noises is the Fourier transform of the signal. We have determined the Barkhausen noise power spectrum \( P(f) \) by using the Fast Fourier Transform of the jumps in magnetization. In our simulations time evolution is taken to be equivalent with the driving rate \( dB \) of the external magnetic field which is considered to be constant in time (adiabatic limit). This approach is acceptable because in simulations equilibrium state is reached in every simulation step, so avalanches cannot overlap in time nor in space. Taking into account this fact the power spectrum is given thus in terms of \( 1/dB \) "frequency" instead of \( 1/t \).

### 5.1.3 Influence of the free parameters

The adjustable parameters of the model are: \( N_s \) - number of spins, \( dB \) - driving rate, \( K \) - demagnetization constant and \( R \) - disorder. The relatively low number of free parameters makes simulation easier, and thus the influence of each parameter can be clearly understood.
A simple test can reveal the role of the parameters and in the same time offers a good check for the validity of the Hamiltonian. It can convince us whether the interactions are taken into account in a proper way.

In the absence of pinning terms and external field the Hamiltonian (5.1) simplifies to:

\[
H = -J \sum_{i=1}^{N_s} S_i S_{i+1} + K \left( \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \prod_{k=i}^{j} \left( \frac{1 + S_i S_k}{2} \right) - \frac{N_s}{2} \right).
\]  

(5.3)

Theoretically one would expect that the system will reach an equilibrium number of domains of equal size, so that the magnetization of the sample under study will be zero. The system governed by the simplified Hamiltonian (5.3) has reached the equilibrium domain number \( n_e \) when:

\[
\frac{dH}{dn_e} = 0.
\]  

(5.4)

In equilibrium all domains have equal length:

\[
l_e = \frac{N_s}{n_e}.
\]  

(5.5)

On the other hand it is always true that

\[
\sum_{j=1}^{n} l_j = N_s.
\]  

(5.6)

Taking into account all the properties expressed through (5.4), (5.5), (5.6) the self-energy term from (5.2) takes the form:

\[
E_{\text{self}} = K \frac{N_s^2}{2n}.
\]  

(5.7)

Given \( n \) domains the Ising-type exchange interaction can be written as:

\[
E_{\text{ex}} = -J(N_s - 1 - 2n).
\]  

(5.8)

The Hamiltonian (5.3) is now written in the form:

\[
H = -J(N_s - 1 - 2n) + K \frac{N_s^2}{2n}.
\]  

(5.9)
Calculating the equilibrium domain number with relation (5.4):

\[ n_e = \frac{N_s^2}{2} \sqrt{\frac{K}{J}}. \quad (5.10) \]

For example: if \( N_s = 2000 \), \( K = 10^{-4} \), \( J = 1 \) then \( n_e = 10 \). Our simulation program reproduces this result correctly.

Parameter \( N_s \) defines the size of the model system, and consequently the equilibrium domain number and size for a given \( K \) value in the absence of external driving magnetic field and pinning effect. From relation (5.10) one can read that increasing the system size will linearly increase the equilibrium number of domains, but the average length of a domain will remain unchanged. This observation is very important, since it allows us to vary \( N_s \) without rescaling parameter \( K \).

Parameter \( K \) characterizes the strength of the magnetic self-energy of the domains, it acts as a demagnetizing factor. The greater \( K \) is the larger the equilibrium domain number is. This parameter also influences the area under the hysteresis loop. Increasing \( K \) the hysteresis loop becomes more narrow. In the absence of pinning centers for high value of \( K \), \( (K = 0.01) \) the hysteresis vanishes. In the other extreme, when \( K \) has very low value \( (K \approx 10^{-5}) \) the system reaches its equilibrium when completely magnetized, and very strong external field is needed to overcome the Ising-type coupling, since the magnetic self-energy term has very small contribution. During the simulations we have set \( K = 10^{-3} \).

Parameter \( dB \) is the driving field rate, defines the step by which the external magnetic field is changed during one simulation step. Since we deal with a quasistatic dynamics, it is recommended to change the external magnetic field intensity with very small steps. We have found that best results are obtained with \( dB = 10^{-3} \).

The ferromagnetic coupling (first) term in the Hamiltonian (5.1) and the external field (third term) favor the creation of a single-domain state. By the other hand, disorder (second term) together with the magnetic self-energy of the domains (last term) favor the multi-domain state. This means that exchange and external driving compete against disorder and self-energy. During the simulations we have fixed the coupling constant \( J = 1 \), and all the quantities are defined according to this convention. Almost all models use this convention, except the random bond Ising model, where the coupling strengths actually contain the randomness, but even though their mean is set to be \(-1\).

Taking into account the above considerations regarding the free parameters of the sys-
Figure 5.1: Hysteresis loops. Parameters: \( N_s = 5000, K = 10^{-3}, dB = 10^{-3}, R = 0.1 \), five successive loops. Inner graph magnifies the indicated region.

tem, simulation results are presented with the following parameter values: \( N_s \) is between 1000 and 10000, \( K = 10^{-3}, dB = 10^{-3} \).

### 5.2 Simulation results

Characteristic **hysteresis loops** are plotted on Fig. 5.1. The shape of the curves fulfills the requirements for the real magnetization phenomena. Many discrete jumps with different sizes are detectable. This means that the model exhibits Barkhausen noise. The hysteresis loops plotted on Fig. 5.1. are obtained by driving the model sample through more (in this very case 5) consecutive complete magnetization–demagnetization cycles while all the simulation parameters were unchanged. One can clearly observe (see inner graph on Fig. 5.1.) that the magnetization does not follow the same path during successive cycles. This fact proves that this model can reproduce the irreversible character of the magnetization–demagnetization process.

A typical **power spectrum** in terms of \( 1/dB \)-type frequency is plotted on Fig. 5.2. Statistics was performed on 1000 configurations. One configuration corresponds to one certain initial random spin- and pinning center arrangement. Compared to experiments every configuration means different ferromagnetic sample (with identical size) with different pinning center distribution. The spectrum shows that our model exhibits \( 1/f \)-type Barkhausen noise with \(-0.91 \) exponent through three decades of data. Power spectrum was computed and averaged over data collected from the whole hysteresis loops (and not one region of it). Here we emphasize again that in our model the frequency is in terms
of $1/dB$. This substitution is allowed because all the simulations were made in the adiabatic limit where the driving rate of the external magnetic field is constant and in every simulation step equilibrium state is reached.

Figure 5.3. shows a comparative plot of \textbf{jump-size distribution} histograms for different system sizes. Jump-size distribution in terms of our model corresponds to the avalanche-size distribution of the experiments, and it is the most relevant distribution function in the study of the BN. From the graph we can read some properties of the distribution function: (i) The jump-size distribution histogram has scaling behavior over more than three decades of data for $N_s = 10000$ with scaling exponent $-1.6$. (ii) No relevant exponential cut-off is detectable. Only the histogram for $N_s = 3000$ exhibits a slight exponential tail. As the system size increases the distribution approaches power-law and the cut-off vanishes. (iii) Since for $N_s = 10000$ we do not detect exponential cut-off anymore, we can affirm that this histogram already indicates the trend that would be followed if the system would be increased to infinity (thermodynamic limit). (iv) The finite size effect has a weak influence on the scaling exponent, since it increases slightly from $-1.7$ (obtained for $N_s = 3000$) until $-1.6$ for $N_s = 10000$. (v) As expected for smaller system size the scaling regime is also shorter. By the other hand for smaller systems less data are obtained because an avalanche cannot be larger than the system itself. For example (read data from Fig. 5.3): in the $N_s = 10000$ case the probability of occurrence for an avalanche that spans over 3000 spins is nonzero, but a same-sized avalanche is impossible to occur in a system that consists of only 3000 spins.
The fact that our simulation results indicate a power-law distribution in the thermodynamic limit means that jump- (avalanche-) sizes exhibit a scale-invariance. This characteristic is expected to be common for all types of crackling noises.

Figure 5.4. (left) shows a comparative plot of signal duration distribution histograms for increasing system sizes. In experiments this quantity measures the length of a Barkhausen jump (measured as a voltage peak induced in the surrounding coil) on time scale. Since in our model the driving rate (step) of the external driving field is equivalent with the time evolution in real systems, signal duration in the model counts the consecutive dB steps (simulation steps) when magnetization jumps occur, i.e. when $\Delta M \neq 0$.

From the graph the following properties can be read: (i) The signal duration distribution histogram has scaling behavior over more than three decades of data for $N_s = 10000$ with scaling exponent $-1.61$. (ii) In the case of small systems ($N_s = 3000$ black circles) a clear exponential cut-off is observable. As the system size is increased the exponential tail reduces, and for $N_s = 10000$ the histogram can be well fitted with pure power-function. This change from power-law with exponential cut-off to simple power-law as the model system’s size increases underlines the generally accepted conception that the cut-off is due to the finite size effect. (iii) For $N_s = 10000$ we do not detect exponential cut-off anymore. This means that in a system with $N_s = 10000$ does not exhibit measurable finite size effect, thus the signal duration histogram indicates the trend that would be followed if the system would be increased to infinity. (iv) In a similar manner like in the case of
the jump-size distribution histogram, finite size effects have a slight influence on the value of the scaling exponent: it changes from $-1.7$ ($N_s = 3000$) to $-1.61$ ($N_s = 10000$).

Figure 5.4. (right) shows a comparative plot of signal area distribution histograms for increasing system sizes. In experiments signal area measures the area under a voltage peak, thus the dimension of this area is Voltage $\times$ Time. In the present model the equivalent quantity is the area under a magnetization jump with length $\Delta B$ (the duration of the signal) and height $\Delta M$. Thus the signal area is a quantity measured in $B \times M$ units. From the graph we can conclude that: (i) The histograms exhibit scaling-law with exponential cut-off over four decades of data. The histogram for $N_s = 10000$ is well fitted with formula: $g(A) = 0.42 \cdot A^{-0.85} \cdot e^{-7 \cdot 10^{-5} A}$. (ii) For smaller system sizes scaling regime is shorter comparative to larger system sizes and exponential cut-off appears sooner and is more relevant. This observation is in accordance with the conception that the cut-off appears due to the finite size effect. (iii) In contrast to the former distribution histograms, the signal area distribution histogram still exhibits exponential tail even for $N_s = 10000$. For larger systems the scaling regime is longer present, the cut-off appears later and becomes less relevant. These results indicate the tendency that in the thermodynamic limit one should obtain pure power-law distributions.
5.3 Stochastic Resonance in the Spin Hamiltonian Model

As it was discussed in the Introduction and in section 3.4, the Ising model at finite temperature driven by a periodic external magnetic field satisfies all the requirements needed for the occurrence of stochastic resonance (SR): it is a bistable system, contains a stochastic force and is exposed to periodic external forcing. In Refs [39, 40, 41, 42] the authors have already reported simulation and analytical calculation results that prove indeed the existence of SR in Ising systems.

Since the spin model with random pinning centers presented in this chapter is a modified Ising-type model, we also examine the possible occurrence of SR in this model.

So far, our model was studied at zero thermodynamic temperature with one-spin-flip equilibrium dynamics. This approach was suitable until we measured the statistical properties of Barkhausen noise in magnetic materials, because most of the ferromagnets are far below their Curie-temperature under standard experimental conditions. In order to model quasistatic driving the equilibrium dynamics was also suitable.

One very important advantage of our spin model compared to the previous spring-block model is that it is very simple to introduce the effect of the positive temperature and to switch from quasistatic driving to nonzero frequency of the external magnetic field.

In the following we present the results concerning SR in the spin model with pinning centers. The dynamics of the system is not anymore a zero-temperature energy minimizing dynamics, but the standard Glauber dynamics defined at a finite temperature $T$

$$W(x, x') = \frac{\exp\left(-\frac{\Delta E}{k_B T}\right)}{1 + \exp\left(-\frac{\Delta E}{k_B T}\right)},$$ (5.11)

where $W(x, x')$ is the transition probability from a spin configuration $x$ to a configuration $x'$ and $\Delta E$ is the energy change associated to this transition. In the case of our single-spin-flip dynamics states $x$ and $x'$ differ only by flipping one certain spin. During the simulations units are defined such that $k_B = 1$.

The external driving magnetic field has the form $B(t) = A \sin(2\pi \nu t)$, where the amplitude is small compared to the ferromagnetic coupling, thus the condition of weak periodic forcing is satisfied. The time unit is defined in Monte Carlo steps (MCS). One MCS in simulations is $N_s$ trials of flipping randomly chosen spins, this corresponds to the average characteristic time $\tau$ necessary for one spin flip caused solely by the thermal fluctuations.
The frequency $\nu$ of the oscillating field is also given in 1/MCS units.

During the simulations we compute the correlation function $\sigma$ between the driving field $B(t)$ and the magnetic response of the system which is the magnetization $M(t)$:

$$\sigma = \frac{\langle B(t)M(t) \rangle}{\sqrt{\langle M(t)^2 \rangle \langle B(t)^2 \rangle}}. \tag{5.12}$$

If we assume that the magnetic response is harmonic as well: $M(t) = M_0 \sin(2\pi\nu t + \varphi)$, where $\varphi$ is the phase-shift with respect to the driving field, the correlation defined in (5.12) becomes after simple algebra:

$$\sigma = \langle \cos \varphi \rangle. \tag{5.13}$$

Based on this assumption we argue that the above correlation function is equivalent up to a constant factor with the experimentally measured dynamical susceptibility: $\chi' = \frac{M_0}{A} \cos \varphi$.

The average in (5.12) is time-average. During the simulation procedure time is measured in MC steps: we collect in every timestep $t_i$ the values of $B(t_i)$ and corresponding $M(t_i)$ and calculate the correlation:

$$\sigma \propto \frac{1}{n} \sum_{i=1}^{n} B(t_i)M(t_i). \tag{5.14}$$

The correlation was studied as a function of temperature $T$, frequency $\nu$, system size $N_s$, driving field amplitude $A$, demagnetizing constant $K$ and disorder $R$.

The area enclosed by the hysteresis loop is strongly related to the energy loss during a magnetization-demagnetization cycle. In experiments for industrial applications it is meaningful to investigate the conditions for minimum energy loss when a ferromagnetic material is exposed to periodic external field. Since industrial processes take place at almost constant temperature (in most cases room temperature) the measurements focus on the effects of the driving frequency on the shape and area of the hysteresis loops at constant temperature (see the results of N. Meyer et al. in section 2.1). If there exists a certain non-trivial driving frequency for which the energy loss has a significant minimum, this would suggest a resonance-like phenomenon as well.

We investigate also the variation of the hysteresis area as a function of driving frequency and temperature in order to find whether there exists an optimal region where the energy loss has a minimum. Unlike in the case when the correlation function was
investigated, in this case the amplitude of the driving field is large enough to drive the sample until saturation.

5.3.1 Results for Stochastic Resonance

The simulation results obtained for the driving field – response correlation indicate clearly the existence of SR. Figure 5.5 shows a characteristic correlation function versus temperature for different values of the driving frequency.

From the correlation curves on the graph we can read several general properties. First, one can observe that for increasing driving frequency the resonance temperature $T_{rez}$ slightly increases. With the parameter set indicated in the caption of Figure 5.5 the resonance temperature varies from 0.35 to 0.85 if the driving frequency $\nu$ varies from $10^{-5}$ to $10^{-2}$. In addition the value of the correlation at the resonance temperature decreases slightly with increasing frequency. This result indicates that as we increase the driving frequency the system’s magnetic response follows less and less the fast changing external field. Despite the fact that there exists an optimal response at a certain resonance temperature even for such high frequencies as 0.01 (1/MCS units), this optimum correlation is less than in the case of slower external driving.

In the followings we will discuss the influence of the amount of quenched disorder ($R$) on the shape of the correlation function.

On Figure 5.6 the parameters are the same as on Figure 5.5, except that the amount
of disorder is reduced to $R = 0.1$. Smaller amount of disorder induces several changes in the behavior of the correlation function. The resonance temperature in the latter case is shifted toward higher temperatures, it varies between 0.75 and 1.45 for frequencies between $10^{-5}$ and $10^{-2}$. The maximum of the correlation function is not influenced by the disorder, it remains around $\sigma_{rez} \approx 0.7$.

The shift of $T_{rez}$ can be understood in a simple way.

Both the presence of pinning centers and the positive temperature act as disorder in the model. Pinning centers define the quenched disorder which does not change in space and time, the effect of the temperature instead results in a dynamic disorder. There is a fundamental difference between the two types of disorder: the pinning centers act as potential barriers and the one-spin dynamics used in the simulations allows a spin-flip only in the case when it is energetically favorable. On the contrary, finite temperature can cause a spin-flip with nonzero probability even if it causes locally energy increase. Due to this property local energy barriers can be overcome with the contribution of temperature and the system can reach new states which would not be accessible with the sole one-spin-flip dynamics.

If there is little amount of quenched disorder, the system needs higher temperature to exhibit SR. This tendency is a property of the Hamiltonian (5.1) which defines the model: due to the pinning (second) term of the Hamiltonian domain boundaries decrease the total energy with $T_i$, where these are normally distributed with standard deviation $R$. For small $R$ higher temperature is required to overcome the ferromagnetic coupling.
Based on these observations we can conclude that the effect of pinning centers in the context of stochastic resonance in this model is to decrease the resonance temperature. Indeed, in the case when $R = 0$ (and $K = 0$ the model reduces to the Ising model) in the small driving frequency limit the resonance temperature $T_{rez} \rightarrow J$, in our simulations $J = 1$ the Ising-type ferromagnetic coupling constant. This result is in agreement with those obtained by Kwan-tai Leung and Zoltán Néda [42].

As it is emphasized on Figure 5.6, when the driving frequency $\nu \geq 0.001$, an additional minor peak appears in the correlation. We argue that this is due to the contribution of the simple Ising model to the SR in this model. This secondary peak is visible at very small temperature ($T = 0.15$) and the corresponding correlation value is strongly dependent on the driving frequency. We have studied the effect of the disorder $R$ and driving frequency on the location of this secondary peak and the corresponding peak-correlation. Comparing Figures 5.5 and 5.6 we can immediately observe that these are sensitive on both the amount of quenched disorder $R$ and the driving frequency.

In Ref. [42] the authors study the phenomenon of SR in Ising model in dimensions one to four. They also report the appearance of a secondary smaller peak in the scaling function in two dimensions. They find that this small peak is slightly below the critical temperature and is observable only for high frequencies.

The analysis of our simulation data yields several results concerning the properties of the small peak. The secondary peak (if observable) shows sensitive dependence on the disorder $R$. If the disorder is large $R = 1$ the secondary peak appears only for the highest frequency $\nu = 0.01$ at $T = 0.25$ and the correlation at this point is 0.55. Decreasing the disorder $R = 0.1$, the secondary peak is clearly observable already for $\nu = 0.001$ (see Fig. 5.6) at $T = 0.15$. It is also visible on Figure 5.6 that the peak-correlation is also very sensitive on the variation of driving frequency: in the case of slower driving ($\nu = 0.001$) the peak correlation is around 0.6, but for $\nu = 0.01$ its value is only around 0.14. Examining carefully the curves for $\nu = 10^{-4}$ and $10^{-5}$ on fig. 5.6 we can observe the "trace" of a possible secondary peak below the resonance temperature with correlation closely as high as at $T_{rez}$. The separation between the resonance temperature and the temperature at which the small peak appears increases with decreasing disorder. If the pinning centers vanish ($R = 0$) the location of the secondary peak is $T \rightarrow 0$ and the resonance temperature $T_{rez} \rightarrow 1$ as already stated above. This observation is in good agreement with the results from Ref. [42], since the critical temperature of the 1D Ising
model is zero.

All the above measurements lead us to conclude that there is a trend followed by the model system: The separation between the two resonance temperatures decreases with increasing disorder and decreasing driving frequency. In the limit of low ($R \to 0$) disorder and high driving frequency the separation is the largest, while in the limit of large disorder and low driving frequency the separation vanishes, there is only one resonance peak detectable. In order to visualize this tendency, we plot three representative cases on Figure 5.7.

The results presented above were obtained for a system size $N_s = 1000$. The next step is to investigate the changes in the B-M correlation if we increase the system size. Figure 5.8 shows a comparative plot of the B-M correlation versus temperature for system sizes in the $1000 \leq N_s \leq 100000$ interval. As indicated in the caption, the driving frequency in this case is $\nu = 0.01$. The reason for using such high frequency is that for large system sizes simulations at low frequency are very time consuming.

The location of the resonance peak is slightly shifted toward higher temperatures for increasing system sizes. The height of the resonance peak is higher in the case of larger systems, however, the height difference is less than 10% (compared to the average peak height of $\approx 0.7$). On the other hand, as one can observe on Figure 5.8 the peak in correlation curve is less sharp in the case of large systems (eg. $N_s = 10^5$) compared to small systems ($N_s = 10^3$), and as a consequence it begins to drop at much larger temperature for big systems. The explanation for the robustness of a large system regarding SR can be
found in the fact that the large system is less sensitive on the same thermal effects as the small system, and there exists a temperature interval \([T_{rez} - 1; T_{rez} + 3]\) (instead of one well-defined temperature value) in which the difference of the B-M correlation from the resonance value is less than 1%. We find this property useful from practical perspective: if a real ferromagnetic material is studied in order to find SR, this could be detected in a temperature interval which is easier to locate than to find a certain temperature value. However, our model is very simplified, it is not suitable for direct comparison with experiments.

Figure 5.9 compares the resonance temperatures for increasing system sizes and driving frequencies. In the case of small systems and low driving frequencies the resonance temperature is almost constant around 0.65-0.75. This is the reason why the symbols on the graph overlap for the frequencies \(\nu = 10^{-5}\) and \(\nu = 10^{-4}\) for \(N_s = 100\) and \(N_s = 1000\). If we increase the number of spins and the driving frequency, the shift of the resonance temperature becomes more relevant. Both parameters, the system size and the driving frequency, have similar effect: large system and fast changing magnetic field move the peak location in the direction of higher temperatures. However, the resonance temperature has a limiting value around \(T_{limit} \approx 2.5\). We consider that the resonance temperature does not diverge to infinity in the thermodynamic limit \((N_s \to \infty)\) for two reasons. (1) SR cannot occur at very high temperature because thermal fluctuations are too high and the system cannot follow the external driving; (2) As it was shown on Figure 5.9 the increase of the driving frequency causes the more emphasized shift of the peak. It is, however, incorrect to go beyond \(\nu = 10^{-2}\), since in this case we would have very small number of
We have also examined the behavior of the correlation function at high temperatures, up to $T = 100$. Our finding is that for high temperatures the correlation decreases according power-law with exponent in the interval $[-1.05; -0.95]$. This exponent is reasonably stable if we vary the most important parameters: system size and driving frequency. On Figure 5.10 we show the correlation function at high temperatures for different system sizes and driving frequencies. The power-law type decrease is in agreement with the result obtained by Z. Néda in Ref. [40], but the values of the exponents do not correspond. In Ref. [40] for the 3D Ising model this exponent is -1.71, indicates a much faster decrease of the correlation for high temperatures.

After showing that SR is present in the model and a peak in the B-M correlation is detectable as a function of temperature, one would expect the same phenomenon as a function of the frequency at fixed temperature. Contrary to our expectation, at any fixed temperature the correlation monotonically decreases for increasing driving frequency. This tendency is shown on Figure 5.11 for different temperature values. It has to be mentioned that this graph contains only several representative “slices”, but in simulations we have swept through a large (temperature – frequency) parameter-space with small steps in each direction. Although the absence of SR as a function of frequency is counter-intuitive, it was forecasted on the correlation versus temperature curves (see for example Figure 5.6). The correlation value for a given temperature always decreases for increasing frequency.
Figure 5.10: B-M correlation at high temperatures for different system sizes and driving frequencies on log-log scale. The power-law with exponent -1 is guide to the eye. Other parameters: A=0.1; R=0.1

Figure 5.11: B-M correlation versus driving frequency for different temperatures. Parameters: Ns=1000; A=0.1; R=0.1
Figure 5.12: (a) Hysteresis loops for $T = 0.5$ and different driving frequencies; (b) Hysteresis loops for $\nu = 10^{-4}$ and different temperatures. Other parameters: $N_s = 1000$, $A = 0.5$, $R = 1.0$

This tendency is not valid in the case of high temperatures where the B–M correlation is very weak for any system size and driving frequency.

For industrial applications it is often needed to investigate the relationship between the microstructure of a sample and its magnetic properties under sinusoidal driving. An example is the study of soft magnetic materials (mostly stainless steel) used for automotive actuation in motors. In his PhD work N. Meyer in Grenoble investigates the 430Nb (17%) stainless steel from Ugitech SA under the conditions of sinusoidal driving of varying frequencies. The goal is to find an optimal driving frequency for which the sample best follows the variation of the external driving field. If one needs in a certain setup a component that works as magnetic gate, this component has to accomplish simultaneously two requirements: (1) produce minimum volumic energy loss during successive magnetization-demagnetization cycles induced by the sinusoidal magnetic field; (2) the sample needs to be sufficiently magnetized in a half-period in order to work properly as a magnetic gate.

The area enclosed by the hysteresis is strongly related to the volumic energy loss through one magnetization cycle. We examined the variation of the hysteresis area for different temperatures and driving frequencies. The results are shown on Figure 5.12.

Figure 5.12 (a) shows the variation of the hysteresis loops for increasing driving frequencies if the temperature of the system is fixed at a small value $T = 0.2$. At this temperature thermal fluctuations are weak, corresponding to standard experimental conditions when ferromagnetic materials are far below their Curie-temperature. It can be seen on the graph that as the driving frequency is increased the area of the hysteresis also
significantly and monotonically grows. Beside the increase of the hysteresis area one can observe that at high frequency ($\nu = 10^{-2}$) the maximum value of the magnetization also slightly decreases. Figure 5.12 (b) shows the evolution of the hysteresis loops for fixed driving frequency and increasing temperature. The frequency was set to $\nu = 10^{-4}$ which corresponds to slow but not quasistatic driving. It can be read from the graph that as the temperature is increased the area enclosed by the loop decreases significantly. This would suggest at first sight that an optimal temperature can be found at which the volumic energy loss during one period is very low. However, this statement is not correct for several reasons. By further increasing the temperature the hysteresis loop becomes very small, almost vanishes. This phenomenon is the consequence of high thermal fluctuations that destroy the ferromagnetic coupling which is responsible for hysteretic behavior of ferromagnets. At very high temperature ($T \geq 1$) the model system has paramagnetic properties. Although the area of the hysteresis decreases for increasing temperature the maximum magnetization drastically drops as an indicator of the paramagnetic behavior. The thermal fluctuations dominate over both the ferromagnetic coupling and the external magnetizing field. This way the sample under study cannot be sufficiently magnetized to work properly as a magnetic gate.

Considering the findings regarding the hysteresis area under the conditions of varying temperature and driving frequency, a redefinition of the total loss is needed. Both the hysteresis area and the maximum magnetization during one cycle are very important in the definition of the loss. Low loss implies thin hysteresis loop and high magnetization at $1/\nu = \pi/2$ in the same time. In the followings the total loss is defined as hysteresis area/maximum magnetization.
On Figure 5.13 the results for the newly defined loss are plotted. The temperature was fixed to $T = 0.2$ in order to be in accordance with experimental conditions: constant low temperature and varying driving frequency. It is clear from the graph that the loss monotonically increases with increasing driving frequency. Even though the loss increases slowly, obeying logarithmic function, one cannot find an optimal driving frequency where the total loss would have a minimum. This result is in agreement with the results of N. Meyer. He also reports monotonically increasing loss (with similar logarithmic shape) and monotonically decreasing field-magnetization correlation for increasing driving frequency at constant room temperature.

5.4 Overview of the Spin Hamiltonian Model with Random Pinning Centers

The simple 1D Ising-type magnetization model presented in this work reproduces successfully on a mesoscopic scale the statistical properties of magnetization phenomena, and in particular the BN.

This model overcomes the main deficiencies of the previous spring-block model: (i) an explicit Hamiltonian governs the dynamics of the system, and (ii) the number of domain walls is not predefined anymore, but domains can appear and disappear depending on which state decreases the total energy of the system. The Hamiltonian of the model accounts for all the relevant interactions that influence the domain-wall dynamics: Ising-type exchange, pinning, interaction with the external field and demagnetization.

The shape of the hysteresis loops obtained with the model is in qualitative agreement with the experimental magnetization curves, and the occurrence of large number of discrete Barkhausen jumps is correctly reproduced. The model captures the main elements of the mesoscopic domain-wall dynamics.

The studied distribution functions of the jump-sizes, durations and areas show scaling behavior over several decades of data. This result is qualitatively in agreement with the experimental results. However, we cannot expect exact numeric correspondence of the critical exponents found in the experiments with those given by the model for several reasons: (1) the experimental results are very sensitive on the special characteristics of the material under study and the experimental setup; (2) this model is a very simple 1D model and its purpose is to reproduce the main general characteristics of magnetic domain-wall dynamics without the intention to focus on special properties of particular
materials.

The present model has two main advantages compared to other well known spin models like RFIM: first, the pinning effect is taken into account realistically in this model since the pinning centers act only on domain boundaries; second, the demagnetization effect is included in the present model, which is very important in real magnetic materials. This property was ignored in numerous previous models, or taken into account in a very complicated manner, so it made the calculations and simulations very difficult.

The effect of the temperature can be simply introduced in the model and treated with the standard Glauber dynamics. On the other hand the relaxation dynamics can be easily changed in a way to simulate the dynamical response of the system under the condition of sinusoidal external driving of varying amplitude and frequency. In this way the possible occurrence of stochastic resonance could be studied in the model. Although, the investigations in this sense are not finalized yet, preliminary results suggest the existence of stochastic resonance as a function of temperature for fixed driving frequency.
Chapter 6

Final Conclusions

Through this work we would like to emphasize the feasibility of very simple models in explaining complex phenomena. Simple models can do a lot! First of all they present the phenomena in a clear, transparent manner. Second, they prove that no complicated models are needed in order to capture the characteristic properties of complex physical phenomena.

In this PhD thesis two simple one-dimensional models were presented which are able to explain the main characteristics of domain-wall dynamics in ferromagnetic materials and can reproduce at least qualitatively the statistical properties of magnetic Barkhausen noise and related phenomena.

The first model is an one-dimensional version of the Burridge-Knopoff type spring-block model, which was originally created to explain the power-law decay of the avalanche-size distributions for earthquakes [43] (empirical Guttemberg-Richter law). However, this model proved to be very useful in describing many, apparently very different natural processes, like: fragmentation of wet granular materials while drying [50], formation of self-assembled nanostructures [51], fracture of glass-plates under mechanical or thermal stress, or even the peeling of an adhesive tape [52]. All these very different examples of applications underline the former statement that very simple models are able to account for the basic description of complex phenomena.

The starting idea for the one-dimensional version of the spring-block model for magnetization phenomena belongs to Professor Yves Bréchet. This is a very simple toy-model based on a mechanical analogy. Despite its simplicity, it is suitable to explain the main features of domain-wall dynamics when a ferromagnetic sample is driven through magnetization cycles. In addition it is able to reproduce the very interesting example of an
athermal phase transition induced by disorder.

The most serious deficiencies of the spring-block model were resolved by a second, Ising-type spin Hamiltonian model. This latter model could also account for the statistical properties of the Barkhausen noise, it could reproduce the scale-invariance of the different representative distributions. Beyond these qualities, the spin Hamiltonian model is also adequate to study the stochastic resonance in the modeled magnetic system under the influence of temperature (stochastic force) and weak periodic external driving.

Both the Ising model and the spring-block type models are very basic and widely applied models in physics. Despite their simplicity, they are able to explain a large number of very different physical processes.

However, numerical correspondence with experimental results cannot be expected from none of the simple models presented in this thesis. Both models are oversimplified and intend only to reproduce the main characteristics of magnetization processes.

We consider that the intention has been realized.
Bibliography


